P systems with elementary active membranes: Beyond **NP** and **coNP**

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Summary

- P systems with active membranes are thoroughly investigated from a complexity-theoretic standpoint
- They have been known to solve **NP** and **coNP** problems in polytime, using elementary division
- We improve this result by solving a **PP**-complete problem

$$\mathbf{PP} \subseteq \mathbf{PMC}_{\mathcal{AM}(-\mathrm{d},-\mathrm{n})}$$

Outline

P systems with elementary active membranes

Recogniser P systems and uniformity

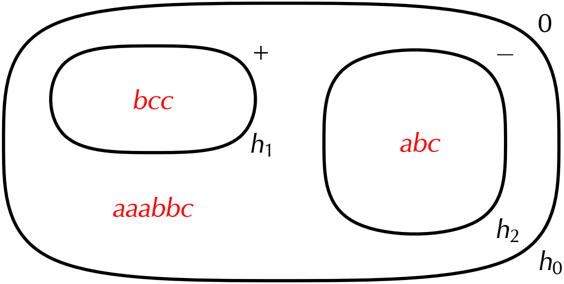
The complexity class **PP**

Solving a **PP**-complete problem

Conclusions and open problems

Membrane structure and its contents

- Membranes have a fixed label and a changeable charge
- The charges regulate which set of rules can be applied
- In each membrane we have the usual multiset of objects



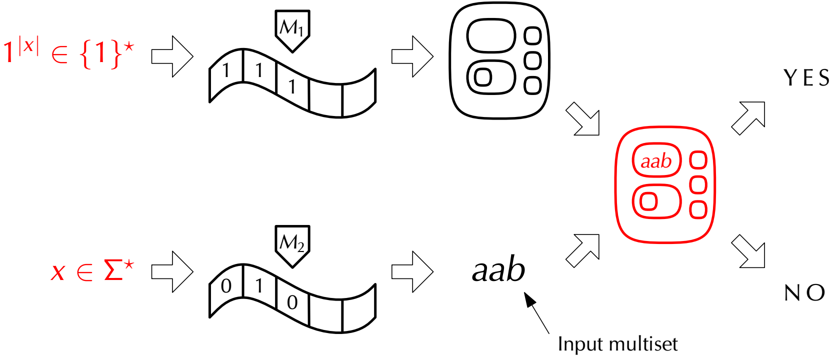
Rules for restricted elementary active membranes

Object evolution	$[a \to w]_h^\alpha$
Send out	$[a]_h^\alpha \to [\ ]_h^\beta\ b$
Send in	$a\ [\ ]_h^\alpha \to [b]_h^\beta$
Elementary division	$[a]_h^\alpha \to [b]_h^\beta\ [c]_h^\gamma$

No dissolution or nonelementary division
Maximally parallel application of rules

Uniform families of recogniser P systems

- For each input length $n = |x|$ we construct a P system Π_n receiving as input a multiset encoding x
- Both are constructed by fixed polytime Turing machines
- The resulting P system decides if $x \in L$



Timeline of P systems with active membranes

- Attacking (and solving) **NP**-complete problems [Păun 1999], uses dissolution and nonelementary division
- Solving **NP**-complete problems [Zandron et al. 2000], no dissolution nor nonelementary division
- Solving **NP**-complete problems [Pérez-Jiménez et al. 2003], uniform, no dissolution nor nonelementary division
- **PSPACE** upper bound [Sosík, Rodríguez-Patón 2007]
- Solving **PP**-complete problems [Alhazov et al. 2009], no nonelementary division, uses either cooperation or postprocessing

The **PP** complexity class

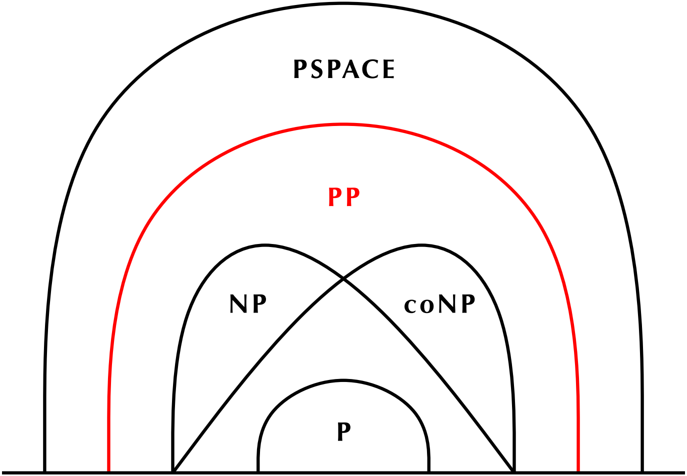
Definition

PP is the class of languages decided by polytime probabilistic Turing machines with error probability strictly less that $1/2$

Definition (equivalent)

PP is the class of languages decided by polytime nondeterministic Turing machines such that more than half of the computations accept

How large is **PP**?



The SQRT-3SAT problem

Problem (SQRT-3SAT)

Given a Boolean formula of m variables in 3CNF,
do more that $\sqrt{2^m}$ assignments satisfy it?

Fact

SQRT-3SAT *is* **PP***-complete*

Encoding SQRT-3SAT instances

- There are $\binom{m}{3}$ sets of 3 variables out of m
- Each variable can be positive or negated (2^3 ways)
- Hence there are $n = 8\binom{m}{3}$ possible clauses
- We can represent a 3CNF formula by an n-bit string
- Checking well-formedness and recovering m from n are easy (polytime)

An example

- If we have 3 variables, the number of clauses is $8\binom{3}{3} = 8$

$$
\begin{array}{lll}
x_1 \vee x_2 \vee x_3 & x_1 \vee x_2 \vee \neg x_3 & x_1 \vee \neg x_2 \vee x_3 \\
x_1 \vee \neg x_2 \vee \neg x_3 & \neg x_1 \vee x_2 \vee x_3 & \neg x_1 \vee x_2 \vee \neg x_3 \\
\neg x_1 \vee \neg x_2 \vee x_3 & \neg x_1 \vee \neg x_2 \vee \neg x_3 &
\end{array}
$$

- Then the formula

$$
\varphi = \underbrace{(x_1 \vee \neg x_2 \vee x_3)}_{\text{3rd}} \wedge \underbrace{(\neg x_1 \vee x_2 \vee \neg x_3)}_{\text{6th}} \wedge \underbrace{(\neg x_1 \vee \neg x_2 \vee x_3)}_{\text{7th}}
$$

is encoded as

$$
\langle \varphi \rangle = 0010\ 0110
$$

A membrane computing algorithm for SQRT-3SAT

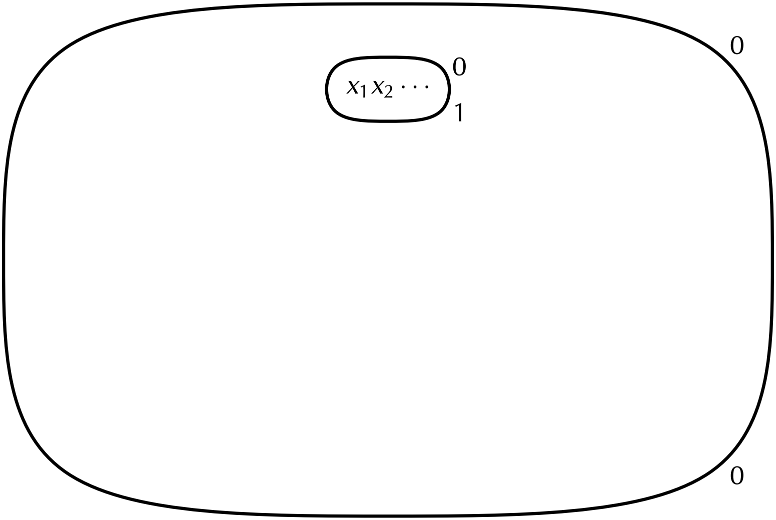
Algorithm

Let φ be a 3CNF formula of m variables

1. Generate 2^m membranes, one for each assignment
2. Evaluate φ in parallel in each of these membranes, send out object t from them if it is satisfied
3. Erase $\lceil\sqrt{2^m}\rceil - 1$ instances of t
4. Output YES if an instance of t remains and NO otherwise

Phase 3 was first proposed by Alhazov et al. 2009 using cooperative rewriting rules

Overview of the computation



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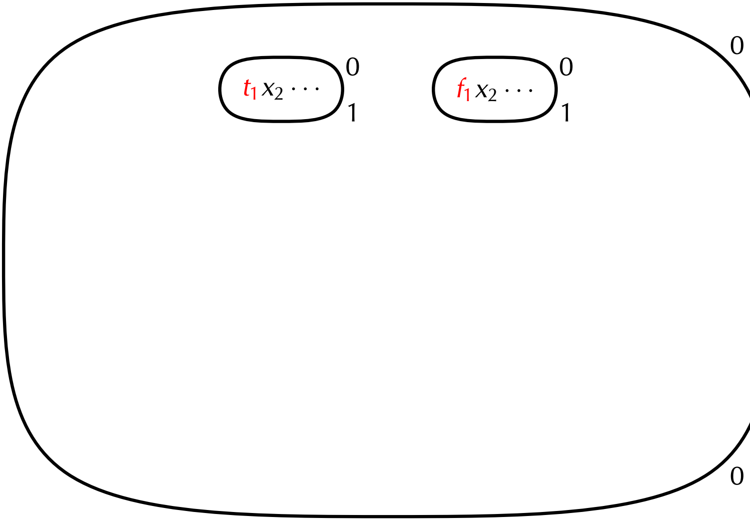

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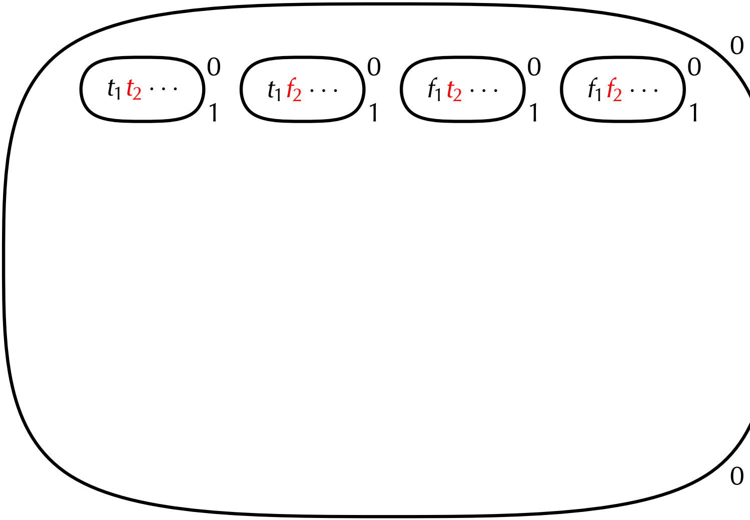

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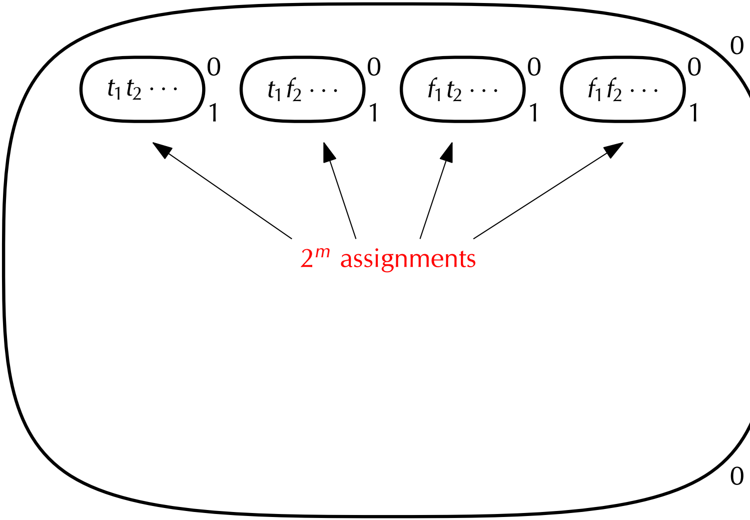

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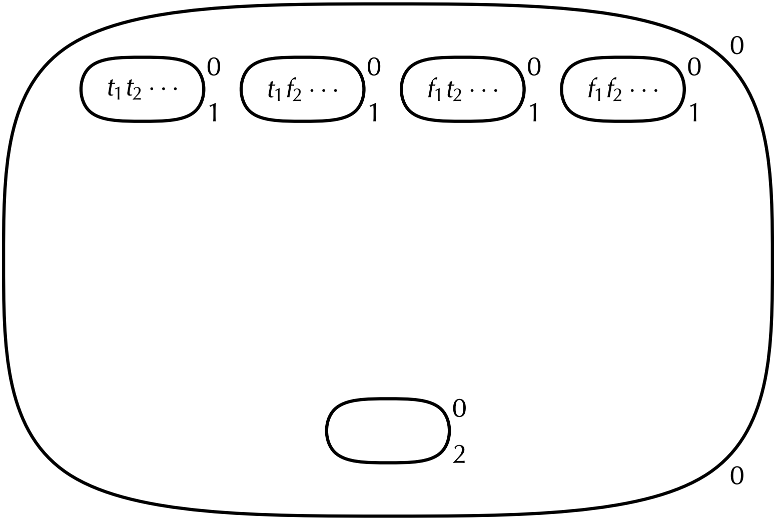

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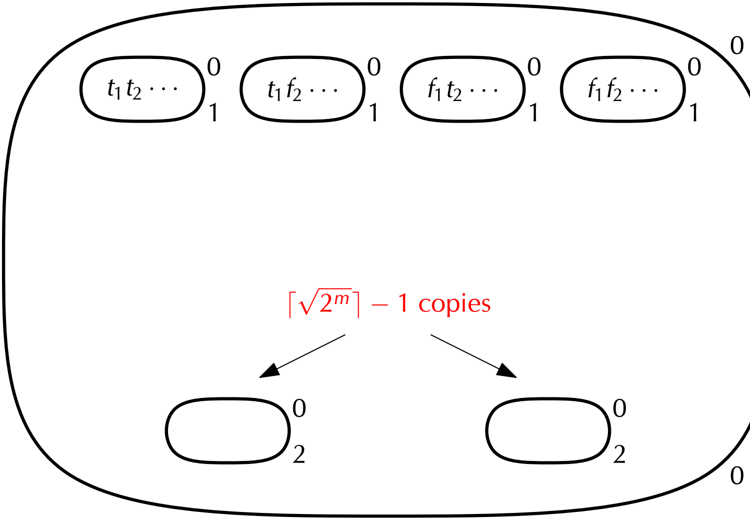

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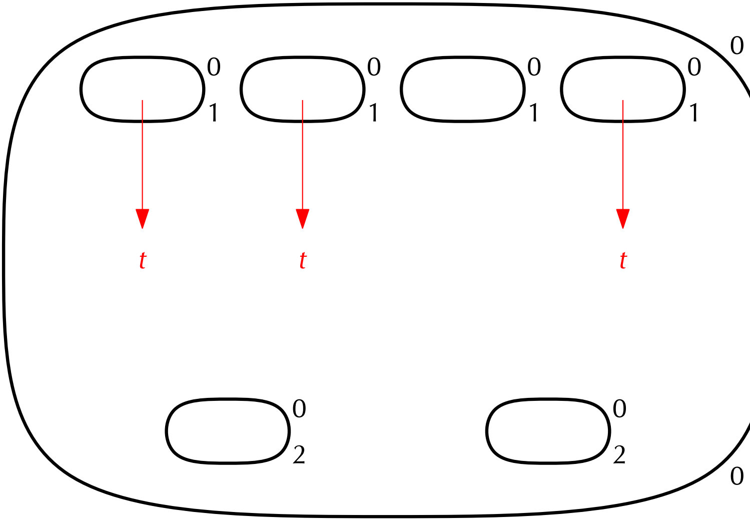

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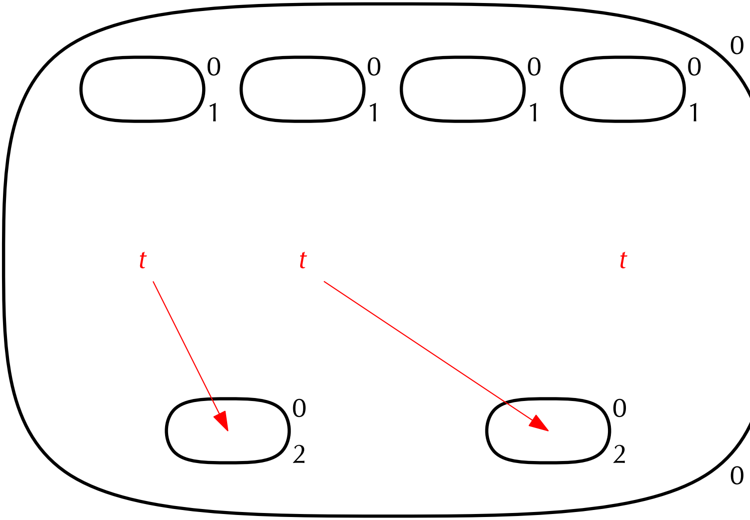

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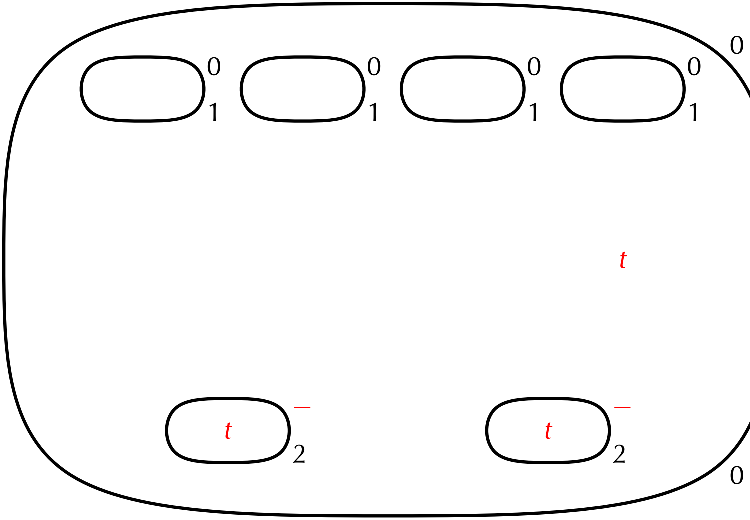

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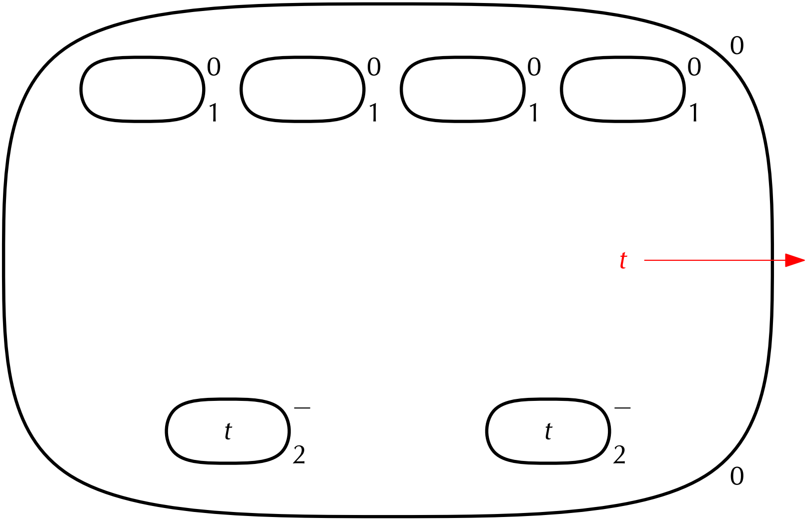

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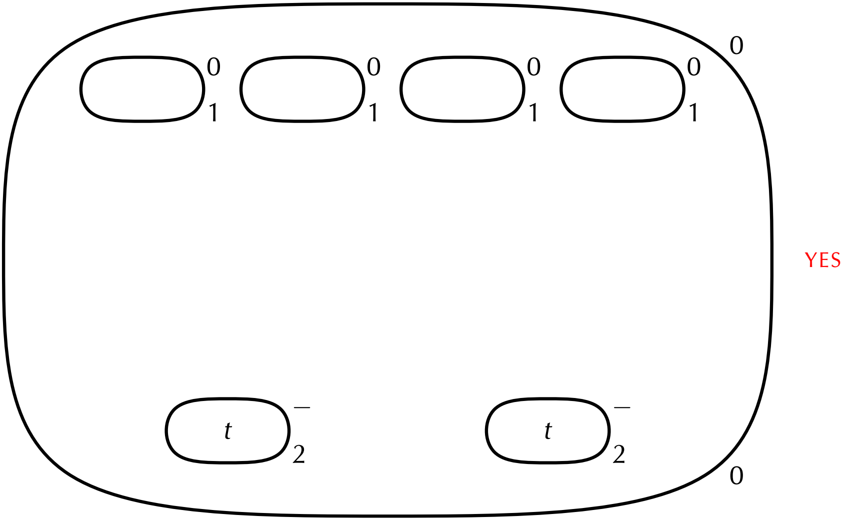


Our main result

Proposition

There is a uniform construction of the family of P systems solving SQRT-3SAT

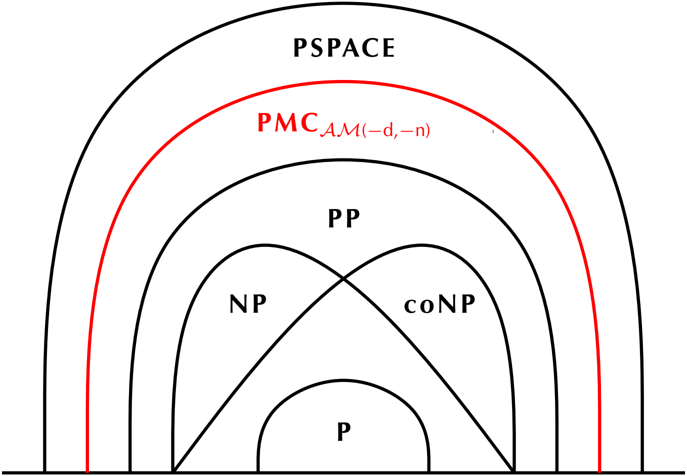
Proposition

SQRT-3SAT $\in \mathbf{PMC}_{\mathcal{AM}(-\mathrm{d},-\mathrm{n})}$

Theorem

$\mathbf{PP} \subseteq \mathbf{PMC}_{\mathcal{AM}(-\mathrm{d},-\mathrm{n})}$

In other words…



Conclusions and open problems

- We solved a **PP**-complete problem in polytime using P systems with restricted active membranes
- As a consequence $\mathbf{PP} \subseteq \mathbf{PMC}_{\mathcal{AM}(-\mathrm{d},-\mathrm{n})} \subseteq \mathbf{PSPACE}$ holds
- However, neither inclusion is known to be strict, and a full characterisation is still missing
- This class is possibly larger than **PP**
- Maybe even $\mathbf{PMC}_{\mathcal{AM}(-\mathrm{d},-\mathrm{n})} = \mathbf{PSPACE}$ holds?

Thanks for your attention!