Flattening the Transition P Systems with Dissolution

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Abstract. Given a transition P system Π with dissolution having several membranes, we construct a P system Π^f having only one membrane and rules involving sets of promoters and inhibitors. The evolution of this "flat" P system Π^f simulates the evolution of initial transition P system Π by replacing any dissolution stage of a configuration in Π by specific rules application in a configuration of Π^f. The transition P systems without dissolution represent a special case of this procedure.

1 Introduction

Membrane systems (also called P systems) represent a biologically inspired model of computation, involving parallel application of rules, communication between membranes and membrane dissolution. Membrane systems are introduced by Gh.Păun and presented in monograph [5] as a class of distributed parallel computing devices inspired by biology. This computing model is represented by complex hierarchical structures with a flow of materials and information which supports their functioning. Essentially, the membrane systems are composed of various compartments with different tasks, all of them working simultaneously to accomplish a more general task.

The motivation of constructing a P system with only one membrane which simulates a P system with multiple membranes and dissolution is to use the former in place of the latter without reducing the generality of a problem. In the conclusion of this paper we discuss the constraints of such a replacement.

We construct the "flat" P system Π^f by replacing objects in membranes of Π with pairs of objects and labels of membranes. Each rule r of Π is translated into sets of rules r^f for Π^f, and dissolution of a membrane labelled by i in Π is translated into the use of rules from a set D_i for Π_f. An evolution step of Π is translated into a single evolution step of Π^f whenever the rules applied in Π do not involve dissolution, and into two evolution steps in Π^f whenever they do. In the second case the first step of Π^f corresponds to rule application in Π and uses only rules from the sets r^f, while the second step corresponds to dissolution of membranes in Π and uses only rules from the sets D_i together with a special rule $\nabla \to 0$ which acts like a semaphore for dissolution.

1.1 P Systems and Multisets

We assume the reader is familiar with membrane computing [5]. For readability, here we present only the necessary notions.

A membrane system consists of a hierarchy of nested membranes, placed inside a distinguishable membrane called *skin*. Each membrane can contain multisets of *objects*, *evolution rules* and other *membranes*.

A *membrane system* of degree m is a tuple $\Pi = (O, \mu, R_1, \ldots, R_m)$, where

- O is an alphabet of objects;
- μ is a membrane structure, with the membranes labelled by natural numbers $1 \ldots m$, in a one-to-one manner;
- each R_i is a finite set of rules associated with the membrane labelled by i; the rules have the form $u \to v$, where u is a non-empty multiset of objects and v a multiset over objects a and messages of the form $(a, out), (a, in_j), \delta$ with the condition that δ can appear at most once;

The rules of the *skin* membrane (which is labelled by 1) do not involve the special symbol δ. This symbol, whenever it is produced by a rule, marks the dissolution of the membrane in which it appears. By dissolution we understand that, after applying rules in a maximally parallel manner, all the objects of a membrane in which δ is produced are sent to its parent membrane. The membrane itself no longer exists, thus modifying the structure of the system.

We can associate *promoters* and *inhibitors* with a rule $u \to v$, in the form $(u \to v)|_{prom, \neg inhib}$, with $prom, inhib$ sets of objects from O. Such a rule associated with a membrane i is applied to a multiset w of objects only if every element of $prom$ is present in w ($w(a) \geq 1, \forall a \in prom$) and no element of $inhib$ is present in w ($w(a) = 0, \forall a \in inhib$). If one or both of the sets $prom, inhib$ is empty, we write the rule $(u \to v)|_{prom, \neg inhib}$ as $(u \to v)|_{prom}$ or $(u \to v)|_{\neg inhib}$ or simply $u \to v$. The promoters and inhibitors of membrane systems formalize the reaction enhancing and reaction prohibiting roles of various substances present in cells [3]. Several papers use rules having at most one object as promoter and one as inhibitor; here we consider sets of promoters and inhibitors. Other generalizations are common in the literature (for instance [1]).

We consider a multiset w over a set S to be a function $w : S \to \mathbb{N}$. When describing a multiset characterized for instance, by $w(s) = 1, w(t) = 2$, $w(s') = 0, s' \in S \backslash \{s, t\}$, we use the representation $s + 2t$. To each multiset w we associate its support, denoted by $supp(w)$, which contains those elements of S which have a non-zero image. A multiset is called non-empty if it has non-empty support. We denote the empty multiset by 0_S or by 0 when the set over which the multiset is defined is clear from the context. We overload the set notation to multisets by using $s \in w$ instead of $w(s) \geq 1$.

The sum of two multisets w, w' over S is the multiset $w + w' : S \to \mathbb{N}, (w + w')(s) = w(s) + w'(s)$. For two multisets w, w' over S we say that w is contained in w' if $w(s) \leq w'(s), \forall s \in S$. We denote this by $w \leq w'$. If $w \leq w'$ we can define $w' - w$ by $(w' - w)(s) = w'(s) - w(s)$.

2 A Simple Semantics of P Systems

We call configuration of a P system a vector of length m (which is the degree of the P system) whose elements are each either a multiset over O or a special symbol $*$. We denote by $\mathcal{C}_{\mathbf{T}}(\Pi)$ the set of configurations for Π. We call intermediate configuration a vector of length m whose elements are either a multiset over $O \cup \{\delta\}$ or the special symbol $*$. We denote by $\mathcal{C}^{\#}_{\mathbf{T}}(\Pi)$ the set of intermediate configurations for Π. Note that $\mathcal{C}_{\mathbf{T}}(\Pi) \subseteq \mathcal{C}^{\#}_{\mathbf{T}}(\Pi)$.

Before we proceed, we introduce some new notations and definitions.

For $W = (w_1, \ldots, w_m) \in \mathcal{C}^{\#}_{\mathbf{T}}(\Pi)$ we denote by $\Delta(W)$ the set of labels $\{i \in \{1, \ldots, m\} / w_i = *\}$ of previously dissolved membranes.

We use $\mu(i) = \mu^1(i)$ to denote the parent of the membrane i with respect to the membrane structure μ. We use $\mu^{k+1}(i)$ to denote the parent of the membrane $\mu^k(i)$. We let μ_W denote the map giving the membrane structure with respect to $W \in \mathcal{C}^{\#}_{\mathbf{T}}(\Pi)$, defined by

$$\mu_W(j) = \begin{cases} \text{undefined, if } j \in \Delta(W) \\ \mu(j), \text{ if } j, \mu(j) \notin \Delta(W) \\ \mu^{l+1}(j), \text{ if } j \notin \Delta(W), \mu(j), \ldots, \mu^l(j) \in \Delta(W), \mu^{l+1}(j) \notin \Delta(W). \end{cases}$$

In other words, $\mu_W(j)$ discards the dissolved membranes which are candidates for parents of j until it reaches an undissolved one and chooses it as the current parent of j.

Definition 1. *We consider a label i and a configuration $W = (w_1, \ldots, w_m)$. A family of multisets $\mathcal{R}_i$ over R_i is called* valid *with respect to W whenever $lhs(\mathcal{R}_i) \leq w_i$ and for each rule $r : (u \to v)|_{prom,\neg inhib}$ such that $r \in \mathcal{R}_i$ we have*

- $w_i(a) \geq 1, \forall a \in prom;$
- $w_i(a) = 0, \forall a \in inhib;$
- *if* $(a, in_j) \in v$ *then* $\mu_W(j) = i.$

A family of multisets of rules $\{\mathcal{R}_j\}_{j \in \{1,\ldots,m\}}$, with each $\mathcal{R}_j$ over R_j, is said to be maximally valid *with respect to a configuration W if it is valid with respect to W and moreover it is maximal with this property: if another family of multisets of rules $\mathcal{R}'_j$ is valid with respect to W and $\mathcal{R}_j \subseteq \mathcal{R}'_j$ then $\mathcal{R}_j = \mathcal{R}'_j$, for all $j \in \{1, \ldots, m\}$.*

We remark that there can exist rules r in R_i producing objects of form (a, in_j) where j is not a child of i in the initial membrane structure μ. Such rules can become active after successive dissolutions.

We define the maximal rewriting stage, which in this semantics includes message sending.

Definition 2. *For $W = (w_1, \ldots, w_m) \in \mathcal{C}_{\mathbf{T}}(\Pi)$ and $V = (v_1, \ldots, v_m) \in \mathcal{C}^{\#}_{\mathbf{T}}(\Pi)$ we define $(w_1, \ldots, w_m) \to_{mpr} (v_1, \ldots, v_m)$ if and only if the following hold:*

– *for all* $i \in \{1,\ldots,m\}\backslash\Delta(W)$ *there exist multisets of rules* $\mathcal{R}_i$ *over* R_i *which are maximally valid with respect to* W *and such that for each* $r \in \mathcal{R}_i$ *and* $(a, in_j) \in rhs(r)$ *we have* $j \notin \Delta(W)$*;*
– *for all* $i \in \{1,\ldots,m\}\backslash\Delta(W)$*,* v_i *is given by* $v_i(\delta) = 1$ *if* $rhs(\mathcal{R}_i)(\delta) > 1$ *and*

$$v_i(a) = w_i(a) - lhs(\mathcal{R}_i)(a) + rhs(\mathcal{R}_i)(a) + \\ + rhs(\mathcal{R}_{\mu_W(i)})(a, in_i) + \sum_{j\in\mu_W^{-1}(i)} rhs(\mathcal{R}_j)(a, out) \tag{1}$$

– *for all* $i \in \Delta(W)$*,* $v_i = *$*.*

We now define the dissolution stage.

Definition 3. *For* $(u_1,\ldots,u_m) \in \mathcal{C}^{\#}_{\mathbf{T}}(\Pi)$ *and* $(v_1,\ldots,v_m) \in \mathcal{C}_{\mathbf{T}}(\Pi)$ *we define* $(u_1,\ldots,u_m) \rightarrow_\delta (v_1,\ldots,v_m)$ *if and only if the following hold:*

– *there is at least one label* i *such that* $u_i(\delta) = 1$*;*
– *let* $U' = (u'_1,\ldots,u'_m) \in \mathcal{C}^{\#}_{\mathbf{T}}(\Pi)$ *be given by* $u'_i = *$ *if* $u_i = *$ *or* $u_i(\delta) = 1$ *and* $u'_i = 0$ *otherwise;*
– $v_i = *$ *if* $u'_i = *$ *and* $v_i = u_i + \sum\{u_j - \delta \mid u'_i \neq *, u_j(\delta) = 1, \mu_{U'}(j) = i\}$ *otherwise.*

We ask that there exists at least one label i such that $u_i(\delta) = 1$ since otherwise there is no need for a dissolution step. We construct U' to provide a "skeleton" for the new membrane structure (note that $\Delta(U) = \Delta(U')$) so we know where to send the contents of the membranes we just dissolved (those j with $u_j(\delta) = 1$), namely to $i = \mu_{U'}(j)$.

We define the transition system $\mathbf{T}$ on $\mathcal{C}_{\mathbf{T}}(\Pi)$ by setting

$$W \Longrightarrow_{\mathbf{T}} V \textbf{ if and only if } W \rightarrow_{mpr} V \text{ or } W \rightarrow_{mpr}\rightarrow_\delta V$$

Example 4. We use a running example of a P system with 4 membranes, with $\mu(4) = 3, \mu(3) = 2, \mu(2) = 1$, with $R_1 = \{r_1 : a \rightarrow (b, in_4), r_2 : b \rightarrow a\}$, $R_2 = \{r_3 : a \rightarrow \delta\}$, $R_3 = \{r_4 : b \rightarrow a\delta\}$ and $R_4 = \{r_5 : b \rightarrow (c, out)$. Consider a configuration $W = (b, a, b, b)$ (see Figure 1).

The evolution of the system, starting from configuration W, is:

$W \rightarrow_{mpr} (a, \delta, a+c+\delta, 0) \rightarrow_\delta (2a+c, *, *, 0) \rightarrow_{mpr} (c, *, *, 2b) \rightarrow_{mpr} (3c, *, *, 0)$

3 Flattening Membrane Systems with Dissolution

Consider a transition P system Π with dissolution; in order to simplify our presentation, we assume its rules do not involve promoters and inhibitors. However the procedure can be applied to transition P systems with promoters and inhibitors. Starting from a system Π, we construct a P system Π^f with only one membrane and with rules involving promoters and inhibitors such that the evolution of Π corresponds to the evolution of Π^f. In more detail, a maximal parallel rewriting stage in Π corresponds to one in Π^f, and a dissolution stage in Π corresponds to a maximal parallel rewriting stage in Π^f.

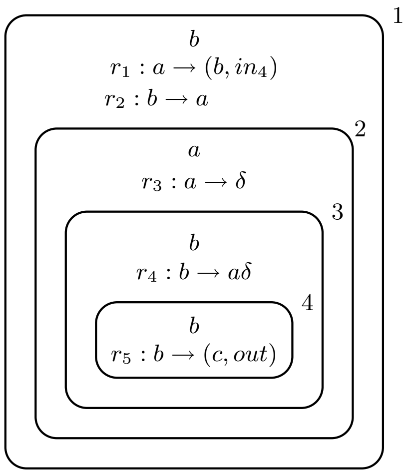

Fig. 1. Membrane structure, rules and the configuration W

Definition 5. *We say that $i \in \{1, \ldots, m\}$ is* dissoluble *if there exists a rule $r \in R_i$ such that $\delta \in rhs(r)$.*

If a membrane is not dissoluble then it will never be dissolved in any possible evolution of any initial configuration. Thus the skin membrane is not dissoluble. On the other hand, if a membrane is dissoluble, then it might be dissolved depending on the initial configuration chosen and on its evolution.

Definition 6. *We define by $maxparent(i)$ the "most remote" membrane which can become a parent of i, after possible dissolutions of membranes. In more detail, $maxparent(i)$ is defined by*

$$maxparent(i) = \mu^k(i) \text{ where } k = \min\{n \geq 1 \mid \mu^n(i) \text{ is not dissoluble}\}.$$

We let $l(i)$ denote k such that $\mu^k(i) = maxparent(i)$.

In **Example 4**, $maxparent(4) = maxparent(3) = maxparent(2) = 1$ and $l(4) = 3$, $l(3) = 2$, $l(2) = 1$. The configuration W is chosen such that membrane 4 does indeed become a child of the skin membrane after one evolution step.

For a multiset w over $O \cup O \times \{in_1, \ldots, in_n\} \cup \{\delta, *\}$ we denote by (w, i) the multiset over $(O \cup \{\delta\}) \times \{1, \ldots, m\}$ obtained by adding to every object $a \in O \cup \{\delta\}$ the label i, replacing in_j by j and replacing $*$ by (δ, i):

- $(w, i)(a, i) = w(a) + w(a, in_i)$;
- $(w, i)(a, j) = w(a, in_j)$, for $j \neq i$;
- $(w, i)(\delta, i) = w(\delta) + w(*)$;
- $(w, i)(\delta, j) = 0$ for $j \neq i$.

Note that by $(w,i)(a,i)$ we understand function application, not string concatenation, since we do not use a string representation for multisets.

The P system Π^f is defined by $\Pi^f = (O^f, \mu^f, R^f)$ with components defined as follows. The alphabet O^f of Π^f is $(O \cup \{\delta\}) \times \{1,\ldots,m\} \cup \{\nabla\}$. The objects (δ, i) are used to represent the fact that the membrane labelled by i is dissolved or undergoing dissolution in Π. The special symbol ∇ is used to separate between the application of rules in Π^f which correspond to rules in Π and the application of rules in Π^f which simulate dissolution in Π. The membrane structure μ^f contains only one membrane. The set R^f of rules consists of a special rule $\nabla \to 0$ together with the union of sets of rules r^f for each rule r of the P system Π and the union of sets of rules D_i for each dissoluble i. Formally,

$$R^f = \bigcup_{\substack{r\in R_i \\ i\in\{1,\ldots,m\}}} r^f \quad \cup \quad \{\nabla \to 0\} \quad \cup \bigcup_{\substack{i\in\{1,\ldots,m\} \\ i \text{ dissolvable}}} D_i$$

Note that the right hand side of the rule $\nabla \to 0$ is the empty multiset 0, meaning that no objects are produced by its application.

We define now the set r_f of rules in Π^f to simulate the application of the corresponding rule r of Π, and the set D_i of rules of Π^f to simulate the dissolution of membrane i in Π. Namely, these rule sets are defined in a manner which ensures that in each evolution step of the “flat” system Π^f we either apply rules from the sets r^f or rules from the sets D_i together with the special rule $\nabla \to 0$.

Definition 7. *For each rule r of Π we define the corresponding set r^f of rules in Π^f. We start by defining $prom(r) = \{(\delta, \mu^l(j)) \mid \exists (a, in_j) \in rhs(r) : \mu^k(j) = i,$ $0 < l < k\}$.*

1. *For each rule $u \to v \in R_i$ such that v contains no out messages and $v(\delta) = 0$, r^f contains only the rule $\underline{r} : (u,i) \to (v,i)|_{prom(r),\neg\{\nabla\}}$;*
2. *for each rule $u \to v \in R_i$ such that v contains no out messages and $v(\delta) = 1$, r^f contains only the rule $\underline{r} : (u,i) \to (v,i)\nabla|_{prom(r),\neg\{\nabla\}}$;*
3. *for each rule $u \to (v, out)w \in R_i$ such that w contains no out messages and $w(\delta) = 0$, r^f is the following set of rules*

   $$r^f = \{\underline{r}_k : (u,i) \to (v, \mu^k(i))(w,i)|_{prom_k(i),\neg inh_k(i)} \mid k \in \{1,\ldots,l(i)\}\},$$

   where the sets of promoters $prom_k(i)$ and inhibitors $inh_k(i)$ are defined by
   - *$prom_1(i) = prom(r)$, $inh_1(i) = \{\nabla, (\delta, \mu(i))\}$;*
   - *$prom_2(i) = prom(r) \cup \{(\delta, \mu(i))\}$, $inh_1(i) = \{\nabla, (\delta, \mu^2(i))\}$;*
   - *…*
   - *$prom_{l(i)-1}(i) = prom(r) \cup \{(\delta, \mu(i)), \ldots (\delta, \mu^{l(i)-2}(i))\}$, $inh_{l(i)-1}(i) = \{\nabla, (\delta, \mu^{l(i)-1}(i))\}$;*
   - *$prom_{l(i)}(i) = prom(r) \cup \{(\delta, \mu(i)), \ldots (\delta, \mu^{l(i)-1}(i))\}$, $inh_{l(i)}(i) = \{\nabla\}$;*
4. *for each rule $u \to (v, out)w \in R_i$ such that w contains no out messages and $w(\delta) = 1$, r^f is the following set of rules*

   $$r^f = \{\underline{r}_k : (u,i) \to (v, \mu^k(i))(w,i)\nabla|_{prom_k(i),\neg inh_k(i)} \mid k \in \{1,\ldots,l(i)\}\},$$

   where $prom_k(i)$ and $inh_k(i)$ are defined as for the previous type of rule.

In **Example 4**, the sets of rules r^f are as follows:

- from rule $r_1 : a \to (b, in_4)$ we get $r_1^f = \{(a,1) \to (b,4)|_{\{(\delta,2),(\delta,3)\},\neg\{\nabla\}}\}$;
- from rule $r_2 : b \to a$ we get $r_2^f = \{(b,1) \to (a,1)|_{\neg\{\nabla\}}\}$;
- from rule $r_3 : a \to \delta$ we get $r_3^f = \{(a,2) \to (\delta,2)|_{\neg\{\nabla\}}\}$;
- from rule $r_4 : b \to a\delta$ we get $r_4^f = \{(b,3) \to (a+\delta,3)|_{\neg\{\nabla\}}\}$
- from rule $r_5 : b \to (c, out)$ we get r_5^f containing the following rules:
  - $(b,4) \to (c,3)|_{\neg\{\nabla,(\delta,3)\}}$;
  - $(b,4) \to (c,2)|_{\{(\delta,3)\},\neg\{\nabla,(\delta,2)\}}$;
  - $(b,4) \to (c,1)|_{\{(\delta,3),(\delta,2)\},\neg\{\nabla\}}$.

The set $prom(r)$ is used to ensure that a rule from r^f can only be applied if, whenever a message (a, in_j) appears in the right hand side of r, the membrane with label j is the current child of membrane i after several dissolutions. This means that if other membranes $\mu^l(j)$ existed between j and $i = \mu^k(j)$, they should be dissolved before r can be applied. Translated to Π^f and r^f, this becomes a requirement for $(\delta, \mu^l(j))$ to be present before the rules from r^f can be applied.

When the right hand side of the rule r does not contain *out* messages, the choice for r^f is straightforward: flatten the multisets in the right and left hand side of r, and if r involves dissolution, we add a special symbol ∇ to the left hand side. The symbol ∇ is also added as an inhibitor to ensure that until ∇ is consumed these rules are not applied. The idea is that ∇ can only be consumed in the stage simulating the dissolution of membranes in Π, which follows whenever ∇ is produced.

When the right hand side of the rule r does contain *out* messages, the rules of the set r^f are defined to ensure that the destination of the messages is the first undissolved parent of i. For example, for $k = 1$, we have in r^f the rule

$$\underline{r}_1 : (u,i) \to (v,\mu(i))(w,i)|_{\neg\{\nabla,(\delta,\mu(i))\}}$$

which "replaces" (v, out) with $(v, \mu(i))$ and can be applied only when $\mu(i)$ is not dissolved (and ∇ is not present). For $k = 2$, we have in r^f a rule

$$\underline{r}_2 : (u,i) \to (v,\mu^2(i))(w,i)|_{\{(\delta,\mu(i))\},\neg\{\nabla,(\delta,\mu^2(i))\}}$$

which can be applied only when $\mu(i)$ is dissolved and $\mu^2(i)$ is not dissolved (and ∇ is not present). Note that $inh_{l(i)}$ contains only ∇ because by definition $\mu^{l(i)}(i) = maxparent(i)$ cannot be dissolved.

Whenever we consider P systems with promoters and inhibitors, we should add the set of promoters and the set of inhibitors of rule r to the set of promoters and the set of inhibitors of each rule in r^f, respectively.

Definition 8. *For each dissoluble label i we define the corresponding set D_i of rules in Π^f as follows:*

$$D_i = \{d_{a,i,k} : (a,i) \to (a,\mu^k(i))|_{prom'_k(i),\neg inh'_k(i)} \mid a \in O, k \in \{1,\ldots,l(i)\}\},$$

where the sets of promoters $prom'_k(i)$ and inhibitors $inh'_k(i)$ are defined by

- $prom'_1(i) = \{\nabla, (\delta, i)\}$, $inh'_1(i) = \{(\delta, \mu(i))\}$;
- $prom'_2(i) = \{\nabla, (\delta, i), (\delta, \mu(i))\}$, $inh'_1(i) = \{(\delta, \mu^2(i))\}$;
- ...
- $prom'_{l(i)-1}(i) = \{\nabla, (\delta, i), \ldots (\delta, \mu^{l(i)-2}(i)\}$, $inh'_{l(i)-1}(i) = \{(\delta, \mu^{l(i)-1}(i))\}$;
- $prom'_{l(i)}(i) = \{\nabla, (\delta, i), \ldots (\delta, \mu^{l(i)-1}(i)\}$, $inh'_{l(i)}(i) = \emptyset$.

The object (δ, i) stands for the fact that the membrane labelled by i is dissolved or undergoing dissolution in Π. If membrane i is dissolved in Π, no objects (a, i) exist in Π^f, and so no rules from D_i can apply. If it undergoes dissolution, then one of the rules in D_i will be applied such that all (a, i) are transformed into objects (a, j), where j is the first undissolved parent of i. The choice for j is made using promoters and inhibitors in a similar manner to the replacement of the message *out* in the sets r^f. The object ∇ is used as promoter to ensure that the rules from D_i are applied only to simulate dissolution in Π, namely, after some object (δ, i) is produced (together with ∇) by some rule from one of the sets r^f of Π^f.

In **Example 4**, the sets D_i are defined for $i \in \{2, 3\}$ and $O = \{a, b, c\}$. The set D_2 contains rules of form $(x, 2) \to (x, 1)|_{\{\nabla, (\delta, 2)\}}$ for $x \in O$. The set D_3 contains rules of form $(x, 3) \to (x, 2)|_{\{\nabla, (\delta, 3)\}, \neg\{(\delta, 2)\}}$ or of form $(x, 3) \to (x, 1)|_{\{\nabla, (\delta, 3), (\delta, 2)\}}$ for $x \in O$.

For an intermediate configuration $W = (w_1, \ldots, w_n)$ of Π let $flat(W)$ denote the configuration of Π^f defined by

- $flat(W)(a, i) = w_i(a)$;
- $flat(W)(\delta, i) = 1$ if $w_i(\delta) = 1$ or $w_i = *$; $flat(W)(\delta, i) = 0$ otherwise;
- $flat(W)(\nabla) = \sum_i w_i(\delta)$.

Note that if W is a configuration, $flat(W)$ does not contain the special symbol ∇.

Theorem 9. *For two configurations W and V of Π,*

$$\textbf{if } W \Longrightarrow_{\mathbf{T}} V \textbf{then } flat(W) \Longrightarrow_{\mathbf{T}} flat(V) \text{ or } flat(W) \Longrightarrow_{\mathbf{T}}\Longrightarrow_{\mathbf{T}} flat(V);$$

$$\textbf{if } flat(W) \Longrightarrow_{\mathbf{T}} flat(V) \textbf{ then } W \Longrightarrow_{\mathbf{T}} V.$$

Proof. Let $W = (w_1, \ldots, w_m)$ and $V = (v_1, \ldots, v_m)$.
First implication:

Firstly, suppose that the transition $W \Longrightarrow_{\mathbf{T}} V$ does not involve dissolution, in other words $W \to_{mpr} V$. We prove that in this case $flat(W) \Longrightarrow_{\mathbf{T}} flat(V)$.

Since $W \to_{mpr} V$, for each $i \in \{1, \ldots, m\}$ there exists a multiset $\mathcal{R}_i$ of rules from R_i such that the family of multisets of rules is maximally valid with respect to W. Moreover, $\Delta(V) = \Delta(W)$ and $lhs(r)(\delta) = 0, \forall r \in \mathcal{R}_i$. We also know that for each $i \in \{1, \ldots, m\} \backslash \Delta(W)$ v_i are obtained from w_i as in Equation (1).

Let $\mathcal{R}$ be the multiset of rules from R^f defined by

- $\mathcal{R}(s) = \mathcal{R}_i(r)$ if $s = \underline{r}$, $r \in R_i$;
- $\mathcal{R}(s) = \mathcal{R}_i(r)$ if $s = \underline{r}_k$, $r \in R_i$ and $\mu^k(i) = \mu_W(i)$;

– $\mathcal{R}(s) = 0$ if $s = \underline{r}_k$, $r \in R_i$ and $\mu^k(i) \neq \mu_W(i)$;

with $\mathcal{R}(s) = 0$ if s is the special rule $\nabla \to 0$ or if s belongs to one of the sets D_i. We prove that $\mathcal{R}$ is maximally valid in $flat(W)$ and that $flat(W) \Longrightarrow_{\mathbf{T}} flat(V)$ by using $\mathcal{R}$.

Note that $lhs(\mathcal{R})(a,i) = (lhs(\mathcal{R}_i), i)$ because when defining $\mathcal{R}$ we always take only one rule from each r^f with the multiplicity that r has in $\mathcal{R}_i$. Hence $\mathcal{R}$ is valid in $flat(W)$. If it was not maximally valid, there would be a rule $s \in R^f$ such that $\mathcal{R}+s$ is valid in $flat(W)$. Since $flat(W)$ does not contain ∇, s cannot be $\nabla \to 0$; nor can it be a rule of form $d_{a,i,k}$ because if $flat(W)$ contains the promoter (δ, i) of $d_{a,i,k}$ then $flat(W)(a,i) = 0, \forall a \in O$. So the rule s would be either $\underline{r}$ or $\underline{r}_k$ for some j and some rule $r \in R_j$. This, however, contradicts the maximal validity of the family $\mathcal{R}_i$ since we would have $lhs(\mathcal{R}_j + r) \leq w_j$.

To prove $flat(W) \Longrightarrow_{\mathbf{T}} flat(V)$ we prove that $flat(V) = flat(W) - lhs(\mathcal{R}) + rhs(\mathcal{R})$. We check the identity for all $(x,i) \in (O \cup \{\delta\} \times \{1,\ldots,m\}$. For all $i \in \Delta(W)$ we have $flat(V)(x,i) = flat(W)(x,i)$ and $lhs(\mathcal{R})(x,i) = rhs(\mathcal{R})(x,i) = 0$. For all $i \in \{1,\ldots,m\} \backslash \Delta(W)$ the identity is inferred from

$$rhs(\mathcal{R})(a,i) = rhs(\mathcal{R}_i)(a) + rhs(\mathcal{R}_{\mu_W(i)})(a, in_i) + \sum_{j \in \mu_W^{-1}(i)} rhs(\mathcal{R}_j)(a, out) \quad (2)$$

which is proved by

$$rhs(\mathcal{R})(a,i) = \sum_j \sum_{r \in \mathcal{R}_j} \sum_{s \in r^f} \mathcal{R}(s) \cdot rhs(s)(a,i) =$$

$$\sum_{j=i} \sum_{r \in \mathcal{R}_j} \mathcal{R}_j(r) \cdot rhs(r)(a,i) + \sum_{j=\mu_W(i)} \sum_{r \in \mathcal{R}_j} \mathcal{R}_j(r) \cdot rhs(r)(a, in_i) +$$

$$+ \sum_{\mu_W(j)=i} \sum_{r \in \mathcal{R}_j} \mathcal{R}_j(r) \cdot rhs(r)(a, out).$$

Secondly, suppose that the transition $W \Longrightarrow_{\mathbf{T}} V$ does involve dissolution, in other words $W \to_{mpr} \to_\delta V$. Then there exists an intermediate configuration $U = (u_1, \ldots, u_m)$ and multisets of rules $\mathcal{R}_i, \forall i \in \{1,\ldots,m\}$ such that $W \to_{mpr} U$ by using $\mathcal{R}_i$ and $U \to_\delta V$. Let $flat(U)$ be the configuration of Π^f corresponding to U.

The proof that $flat(W) \Longrightarrow_{\mathbf{T}} flat(U)$ is similar to the proof that $W \Longrightarrow_{\mathbf{T}} V$ implies $flat(W) \Longrightarrow_{\mathbf{T}} flat(V)$. Moreover, we note that $flat(U)(\nabla) > 0$. We prove now that $flat(U) \Longrightarrow_{\mathbf{T}} flat(V)$. Let $\mathcal{D}$ be the multiset of rules over R^f defined by $\mathcal{D}(s) = 0, \forall s \in r^f$, $\mathcal{D}(\nabla \to 0) = flat(U)(\nabla)$ and

– $\mathcal{D}(d_{a,i,k}) = 0$ if $flat(U)(\delta, i) = 0$ or $\mu^k(i) \neq \mu_V(i)$;
– $\mathcal{D}(d_{a,i,k}) = flat(U)(a,i)$ if $flat(U)(\delta,i) > 0$ and $\mu^k(i) = \mu_V(i)$.

We prove that $\mathcal{D}$ is maximally valid in $flat(U)$ and that $flat(U)$ evolves to $flat(V)$ using $\mathcal{D}$. The validity of $\mathcal{D}$ follows from its definition. To see that it is also maximal with this property, suppose there exists $s \in R^f$ such that $\mathcal{D} + s$

is valid in $flat(U)$. Clearly s cannot be a rule from one of the sets r^f since all those rules are inhibited by ∇. Also, s cannot be the special rule $\nabla \to 0$ because $\mathcal{D}(\nabla \to 0) = flat(U)(\nabla)$. Suppose s is one of the rules $d_{a,i,k}$. If $flat(U)(\delta, i) > 0$ this contradicts $\mathcal{D}(d_{a,i,k}) = flat(U)(a, i)$. If $flat(U)(\delta, i) = 0$ then the promoter (δ, i) of $s = d_{a,i,k}$ is missing from $flat(U)$.

To prove that $flat(U)$ evolves to $flat(V)$ using $\mathcal{D}$, we show that $flat(V) = flat(U) - lhs(\mathcal{D}) + rhs(\mathcal{D})$. To this purpose, note that $flat(V)(a, i) = 0 = flat(U)(a, i) - lhs(\mathcal{D})(a, i)$ and $rhs(\mathcal{D})(a, i) = 0$ for all $a \in O$ and i such that $(\delta, i) \in flat(U)$. Moreover, for those i such that $flat(U)(\delta, i) = 0$ we have $flat(V)(a, i) = flat(U)(a, i) + rhs(\mathcal{D})(a, i)$ and $lhs(\mathcal{D})(a, i) = 0$, $\forall a \in O$.

Second implication:

First suppose that $flat(W) \Longrightarrow_{\mathbf{T}} flat(V)$ by using a multiset $\mathcal{R}$ of rules over R^f. Since $flat(W)$ does not contain ∇, the multiset $\mathcal{R}$ of rules cannot contain rules from the sets D_i nor the special rule $\nabla \to 0$. Moreover, it cannot contain rules which have (δ, i) in the right hand side (because (δ, i) is always accompanied by ∇). From the way the sets r^f are defined, a valid multiset of rules $\mathcal{R}$ cannot contain two distinct rules from the same set r^f. Thus we can define the multisets $\mathcal{R}_i$ over each set R_i in Π by $\mathcal{R}_i(r) = \mathcal{R}(s)$ if there exists $s \in r^f$ such that $\mathcal{R}(s) > 0$ (if it exists, s is unique) and $\mathcal{R}_i(r) = 0$ otherwise.

We prove that the family of multisets $\mathcal{R}_i$ of rules is maximally valid with respect to W and that V is obtained from W by using it. For validity, it suffices to see that $lhs(\mathcal{R}_i) \leq w_i$ and that if $(a, in_j) \in rhs(r)$ then we obtain that $\mu_W(j) = i$, because $flat(W)(\delta, \mu^s(j)) = 1$ for all $(\delta, \mu^s(j)) \in prom(r)$. For maximal validity, suppose there exists some family of multisets $\mathcal{R}'_i$ valid with respect to W such that $\mathcal{R}_i \leq \mathcal{R}'_i$. Then the multiset $\mathcal{R}'$ of rules defined as in the proof of the first implication (on page 8) is valid with respect to $flat(W)$ and $\mathcal{R} \leq \mathcal{R}'$ which implies that $\mathcal{R}_i = \mathcal{R}'_i$. To see that V is obtained from W by using $\mathcal{R}_i$ we just use identity (2) which holds for the multisets $\mathcal{R}$ and $\mathcal{R}_i$ defined above.

Proposition 10. *For two configurations W and V of Π and a configuration X of Π^f such that $\nabla \in X$,*

$$\textbf{if } flat(W) \Longrightarrow_{\mathbf{T}} X \Longrightarrow_{\mathbf{T}} flat(V) \textbf{ then } W \Longrightarrow_{\mathbf{T}} V.$$

Proof. Let $W = (w_1, \ldots, w_m)$ and $V = (v_1, \ldots, v_m)$.

Let $\mathcal{R}$ be the multiset of rules used in $flat(W) \Longrightarrow_{\mathbf{T}} X$. Since $\nabla \notin flat(W)$ it follows (exactly as in the proof of the second implication of Theorem 9) that $W \to_{mpr} U$ for a intermediate configuration U. Moreover, we obtain that $flat(U) = X$.

Let $\mathcal{D}$ be the multiset of rules used in $X \Longrightarrow_{\mathbf{T}} flat(V)$. Since $\nabla \in X$, any rule in $\mathcal{D}$ is either from the sets D_i or the special rule $\nabla \to 0$. Clearly, $\mathcal{D}(\nabla \to 0) = X(\nabla)$. Moreover, for each i such that $(\delta, i) \in X$ and for each a such that $(a, i) \in X$, there can be exactly one $k = k(i) \leq l(i)$ such that $\mathcal{D}(d_{a,i,k}) > 0$. If either (δ, i) or (a, i) does not appear in X, then $\mathcal{D}(d_{a,i,l}) = 0$ for all $l \leq l(i)$.

We prove that $U \rightarrow_\delta V$. Consider $U' = (u'_1, \ldots, u'_m)$ as in Definition 3. Then for $k(i)$ previously defined we have $\mu^{k(i)}(i) = \mu_{U'}(i)$ (because $X = flat(U)$). Thus for all $j \notin \Delta(V)$ we have

$$v_j(a) = flat(V)(a, j) = X(a, j) + \sum_i \{X(a, i) \mid (\delta, i) \in X, \mu_{U'}(i) = j\}$$

which implies

$$v_i = u_i + \sum \{u_j - \delta \mid u'_i \neq *,\, u_j(\delta) = 1,\, \mu_{U'}(j) = i\},$$

and so concluding the proof.

For P systems without dissolution, Theorem 9 and Proposition 10 can be combined into a single result.

Corollary 11. *Let Π be a P system without dissolution and Π^f its associated P system with only one membrane. For W and V configurations of Π,*

$$W \Longrightarrow_{\mathbf{T}} V \textbf{ if and only if } flat(W) \Longrightarrow_{\mathbf{T}} flat(V).$$

Remark 12. We end by emphasizing the size of the P system Π^f with respect to that of Π. Thus, the cardinality of the alphabet O^f is,

$$card(O^f) = (card(O) + 1) \cdot m + 1$$

while the cardinality of the rule set R^f is, according to Definitions 7 and 8,

$$card(R^f) = \sum_{i \in \{1,\ldots,m\}} \sum_{r \in R_i} (card\{r \in R_i \mid \nexists (a, out) \in rhs(r)\} +$$

$$+ l(i) \cdot card\{r \in R_i \mid \exists (a, out) \in rhs(r)\}) + \sum_{i \text{ dissolvable}} l(i) \cdot card(O) + 1$$

4 Conclusion

In this paper we present a general approach for P systems with dissolution, based on the use of special symbols as promoters and inhibitors. The main result is Theorem 9, where we prove that the evolution of each transition P system Π with multiple membranes is simulated by the evolution of its "flat" counterpart Π^f. This result is a generalization of the existing construction for P systems without dissolution, as can be seen in Corollary 11.

The results presented here may be used to simplify proofs of statements involving general transition P systems by using only P systems with one membrane. However, a caveat applies: the evolution in Π^f is staggered with respect to the evolution in Π since two steps will take place in Π^f for one involving dissolution on Π. Other concerns may appear regarding the increasing number of objects and rules in the P system Π^f, according to Remark 12.

The idea of using a single membrane system to simulate P systems with multiple membranes has previously appeared in several papers. A formal presentation for (tissue) P systems without dissolution can be found in [4].

An early paper dealing with dissolution is [6]. While the paper is not directly concerned with the simulation of a multiple membrane system by a one membrane system, it presents the encoding of a multiple membrane system with dissolution into a particular kind of Petri net. The resulting Petri net has transitions which simulate rule applications and special transitions which simulate objects passing from dissolved membranes to their parents. In terms of Example 4, these special transitions simulate rules $(x, 3) \rightarrow (x, 2)$ and $(x, 2) \rightarrow (x, 1)$ for $x \in \{a, b\}$. The authors do not explain in sufficient detail the semantics of their version of Petri nets, and do not treat the case of simultaneous dissolutions. More precisely, the Petri net simulating Example 4 should have three phases in the "macro-step" in order to properly simulate the evolution of the system: one for simulating maximally parallel rule application, one for moving objects from the dissolved membrane 3 to membrane 2 and one for moving objects from the dissolved membrane 2 to membrane 1.

A recent paper presenting a flat form for P systems is [2]. The construction of this paper depends on the use of a special syntax and semantics for P systems, named *P algebra*. This semantics, while complicated, is useful in establishing various behavioural equivalences.

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