

Spiking Neural P Systems with Neuron Division

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efficient solutions to hard problems
eg. SAT

- nondeterminism
- exponential size resources (*precomputed*)

versus

- deterministic
- polynomial time
- polynomial size system (*initially*)

GhP: ‘linear parallel time’

'PPP' previous work

L. Pan, G. Păun, M.J. Pérez–Jiménez.

Spiking neural P systems with neuron division and budding.

7th Brainstorming Week on Membrane Computing, vol. II,
pp. 151–168 (2009)

goal uniform solution to SAT

cf. active membranes (using spiking)

instance given as 'input'

size of problem as parameter

precomputed: polynomial size system

here: same methodology as PPP–paper

but – do not use budding

and – constant size initial system (vs. linear)

spiking neural P system with neuron division

neurons	nodes	'places'
synapses	directed edges	
spikes	objects	'tokens'

[$E / a^c \rightarrow a^p ; d$]

extended firing

E 'test' regular expression over a
 $c \geq 1, p \geq 0, c \geq p$
consume, produce
 $d \geq 0$ *delay* 'blocks'

parallelism – one rule for each neuron
(non)determinism

initial system

depends on size

Input module

$$a^4$$

$$a^4/a^3 \rightarrow a^3; 4n$$

$$a \rightarrow a; nm-1$$

d_0

initial contents

$$[a^2]_{b_1} \rightarrow []_{d_1} || []_{b_2}$$

b_1
label

$$[a^2]_{e_1} \rightarrow []_{c_{x_1}} || []_{e_2}$$

e_1

Satisfiability checking module

deterministic

$$[a^2]_{g_1} \rightarrow []_{h_1} || []_{g_2}$$

g_1

forgetting

$$[a^2]_{f_1} \rightarrow []_{t_1} || []_{f_2}$$

f_1

$$a^7$$

$$a^7/a^2 \rightarrow a^2; 2n-3$$

$$a^5/a^2 \rightarrow a^2; 2n-1$$

$$a^3 \rightarrow a^3; nm+2$$

3

$$a^2$$

$$a \rightarrow a$$

$$a^2 \rightarrow a^2$$

$$a^3 \rightarrow \lambda$$

$$a^4 \rightarrow a$$

2

$$a^2 \rightarrow \lambda$$

$$a^3 \rightarrow a; 1$$

$$a^6 \rightarrow a^2; 1$$

4

$$a \rightarrow a$$

$$a^2 \rightarrow a^2$$

1

$$(a^2)^+ / a \rightarrow a$$

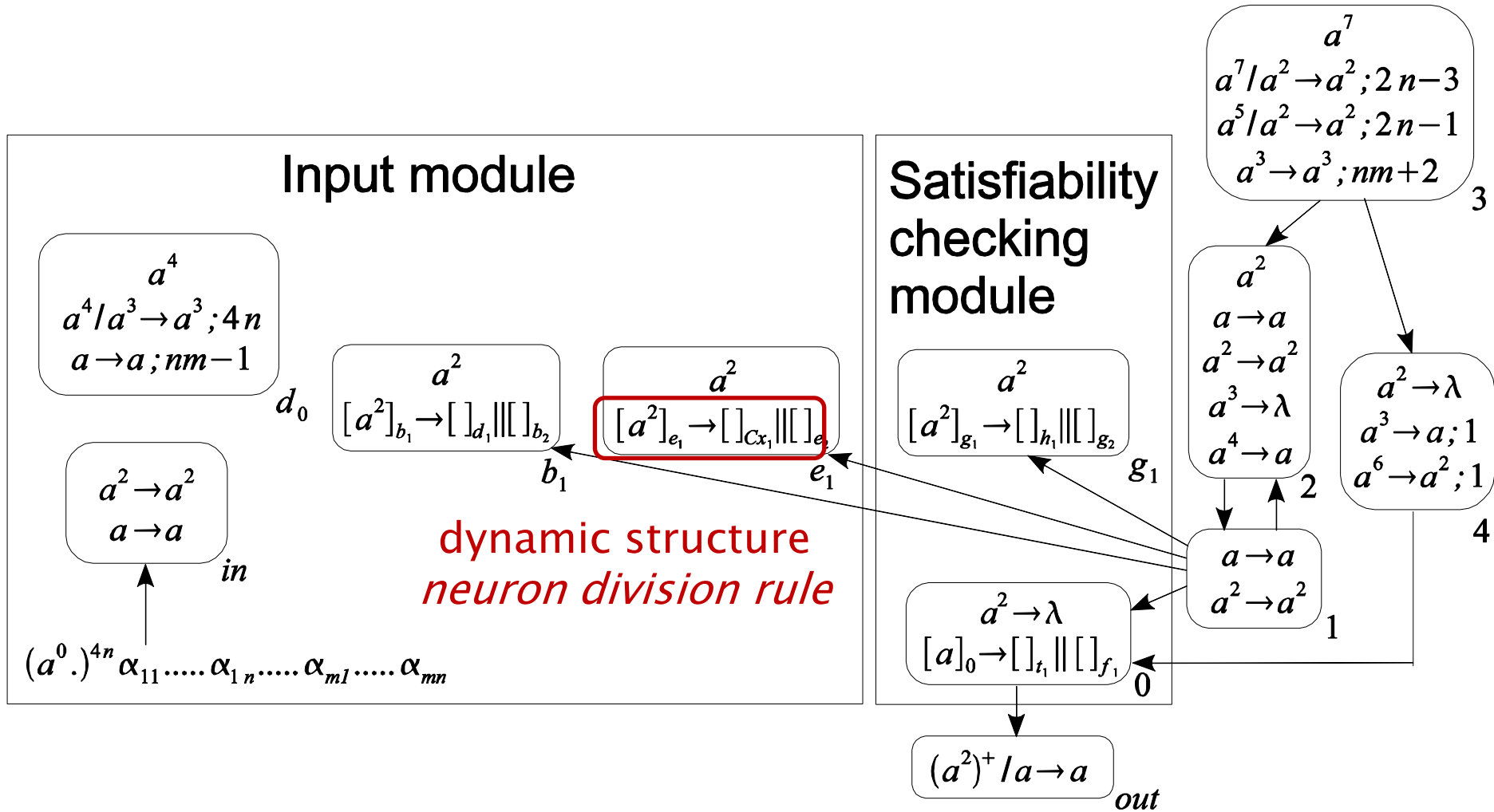
regular!

out output

input: problem instance
spike train

$$(a^0 \cdot)^{4n} \alpha_{11} \dots \alpha_{1n} \dots \alpha_{m1} \dots \alpha_{mn}$$

initial system



neuron division

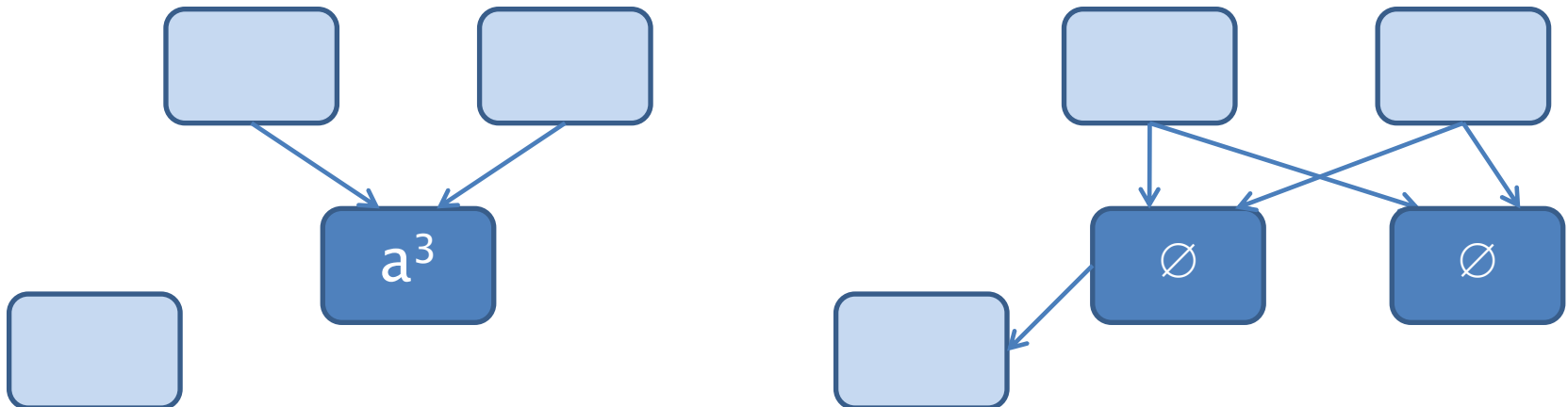
$$[E]_i \rightarrow []_j \parallel []_k$$

neuron division

E regular expression 'test'

children inherit synapses
+ *synapse dictionary* (based on label)

no initial spikes



solving SAT

uniform encoding, parameters:
n variables and m clauses

SAT(n,m)

$X = \{ x_1, x_2, \dots, x_n \}$ variables

$\gamma = C_1 \wedge C_2 \wedge \dots \wedge C_m$ proposition
conjunction 'and' of

C_j clause,

disjunction 'or' of

x_i or $\neg x_i$ literals

$$\underset{1}{(x_1 \vee x_2 \vee x_3)} \wedge \underset{1}{(\neg x_2 \vee x_4)} \wedge \underset{0}{(\neg x_1 \vee x_3 \vee \neg x_4)} \underset{?}{\quad} \underset{1}{\quad}$$

satisfiability – assignment to variables

system strategy

closely follows Pan, Păun, Pérez–Jiménez

Generation Stage

generates structure of system
division

Input Stage

reads instance γ into system

Satisfiability Checking Stage

test all valuations on clauses

Output Stage

any valuation left?

input encoding

$X = \{ x_1, x_2, \dots, x_n \}$ variables

$Y = C_1 \wedge C_2 \wedge \dots \wedge C_m$ clauses

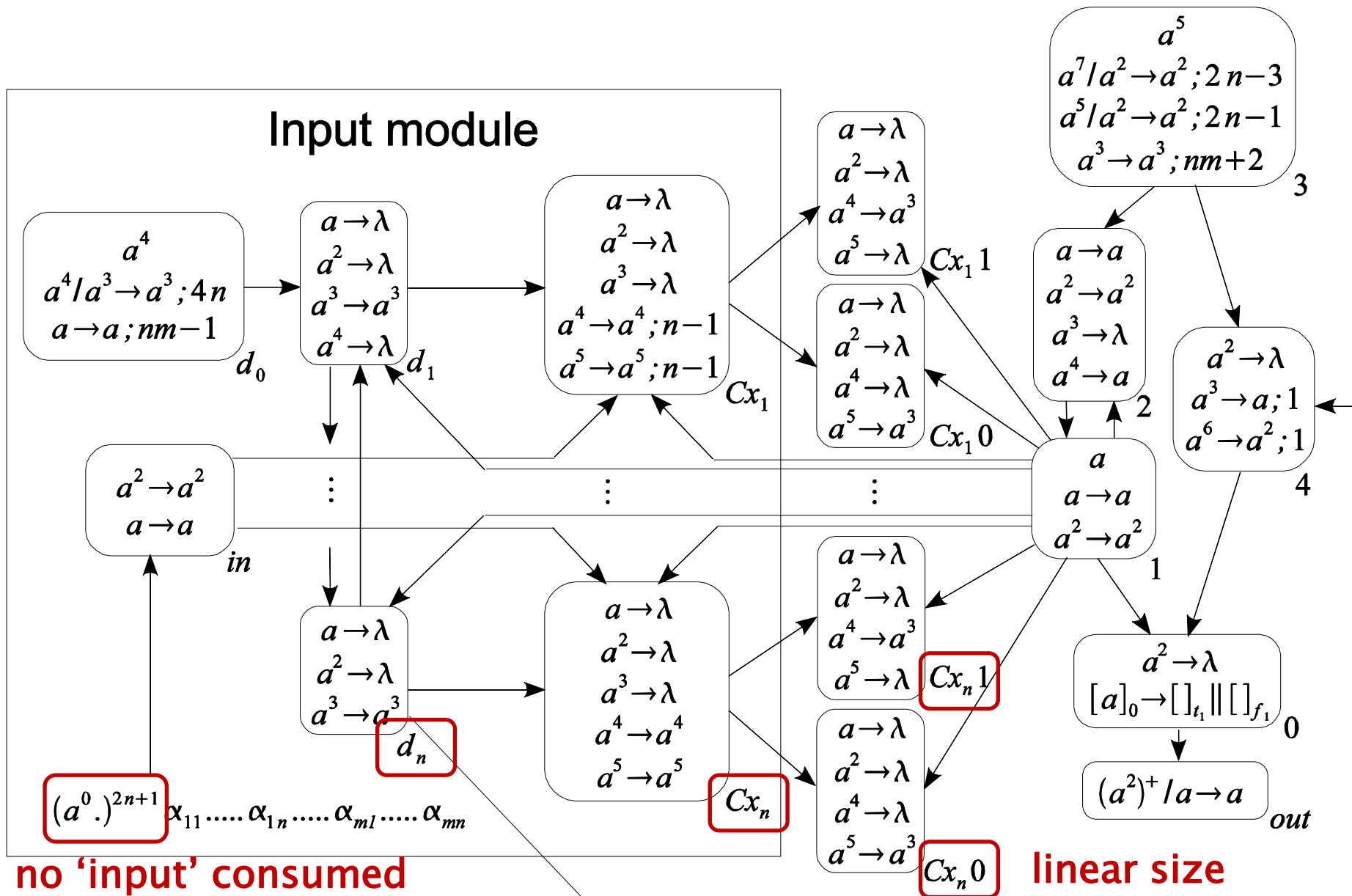
$$\alpha_{ij} = \begin{cases} a & x_j \text{ occurs in } C_i \\ a^2 & \neg x_j \text{ occurs in } C_i \\ a^0 & \text{otherwise} \end{cases}$$

spike train, sent into initial neuron

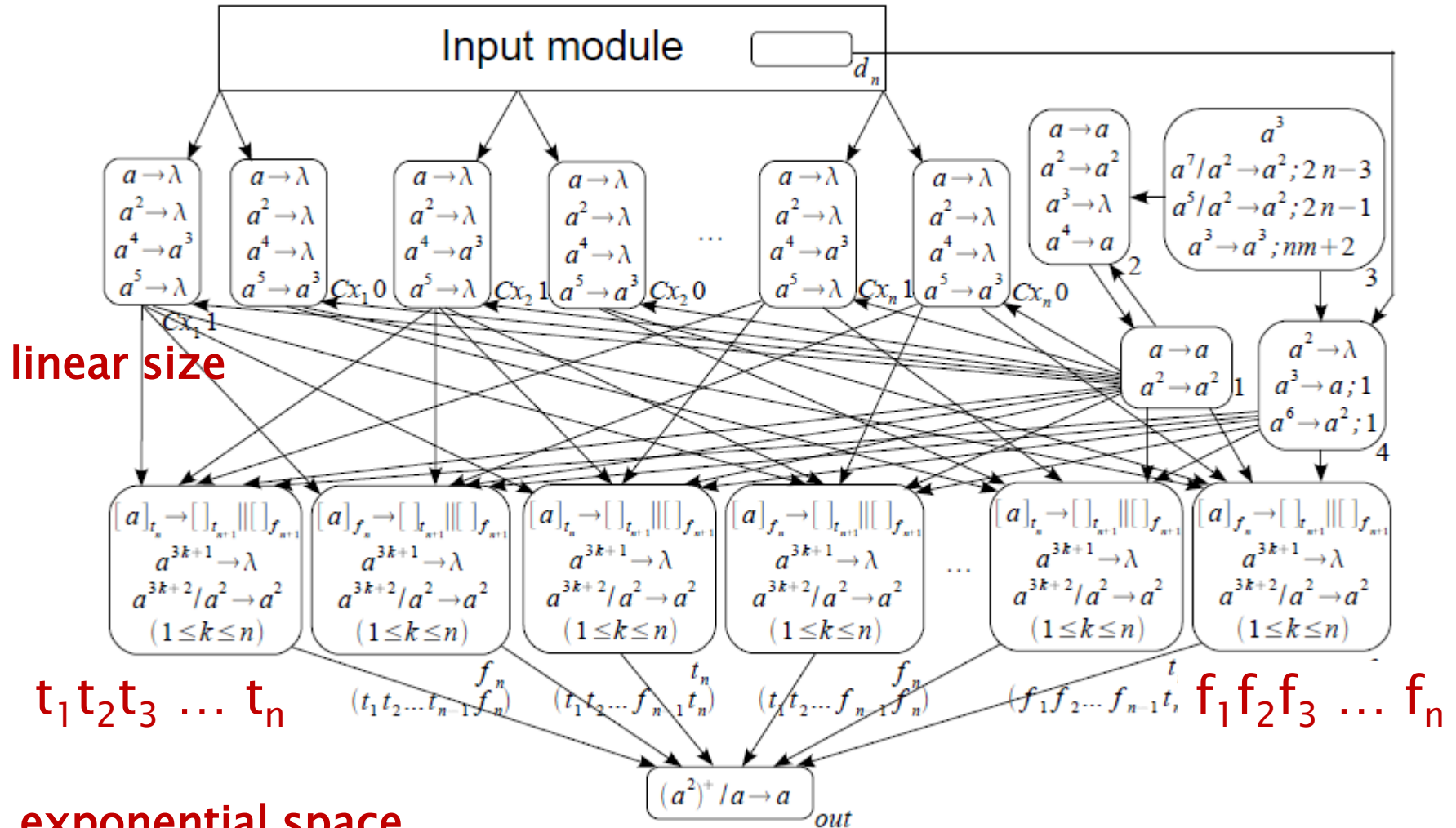
$$(a^0)^{4n} \alpha_{11} \dots \alpha_{1n} \dots \underbrace{\alpha_{i1} \alpha_{i2} \dots \alpha_{in}}_{\text{clause } C_i} \dots \alpha_{i1} \dots \alpha_{in}$$

delay

generation: step $2n-1$



generation: step $4n-1$



exponential space
all possible valuations

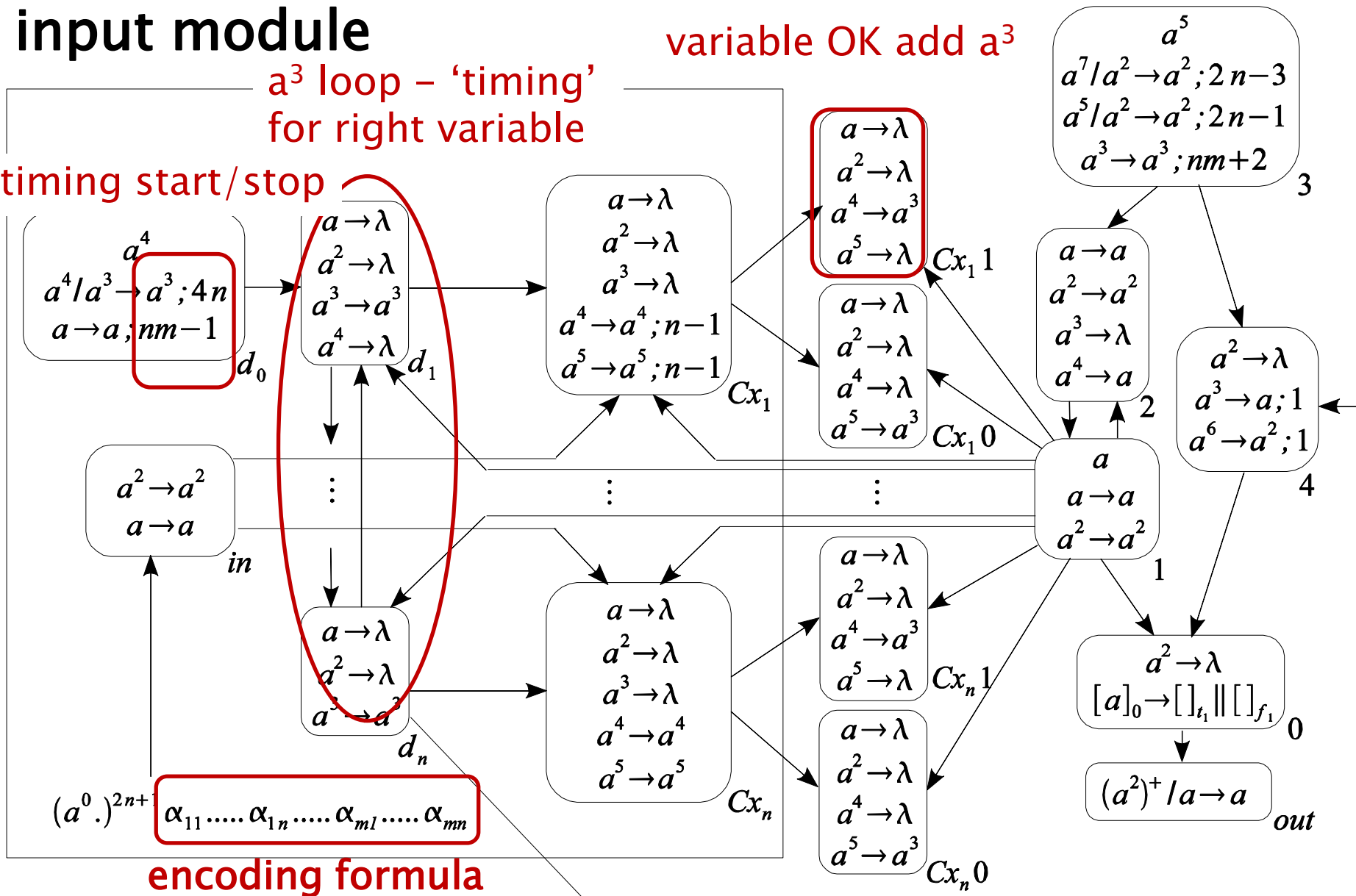
input: step $4n+1 \dots 4n+nm+1$

input module

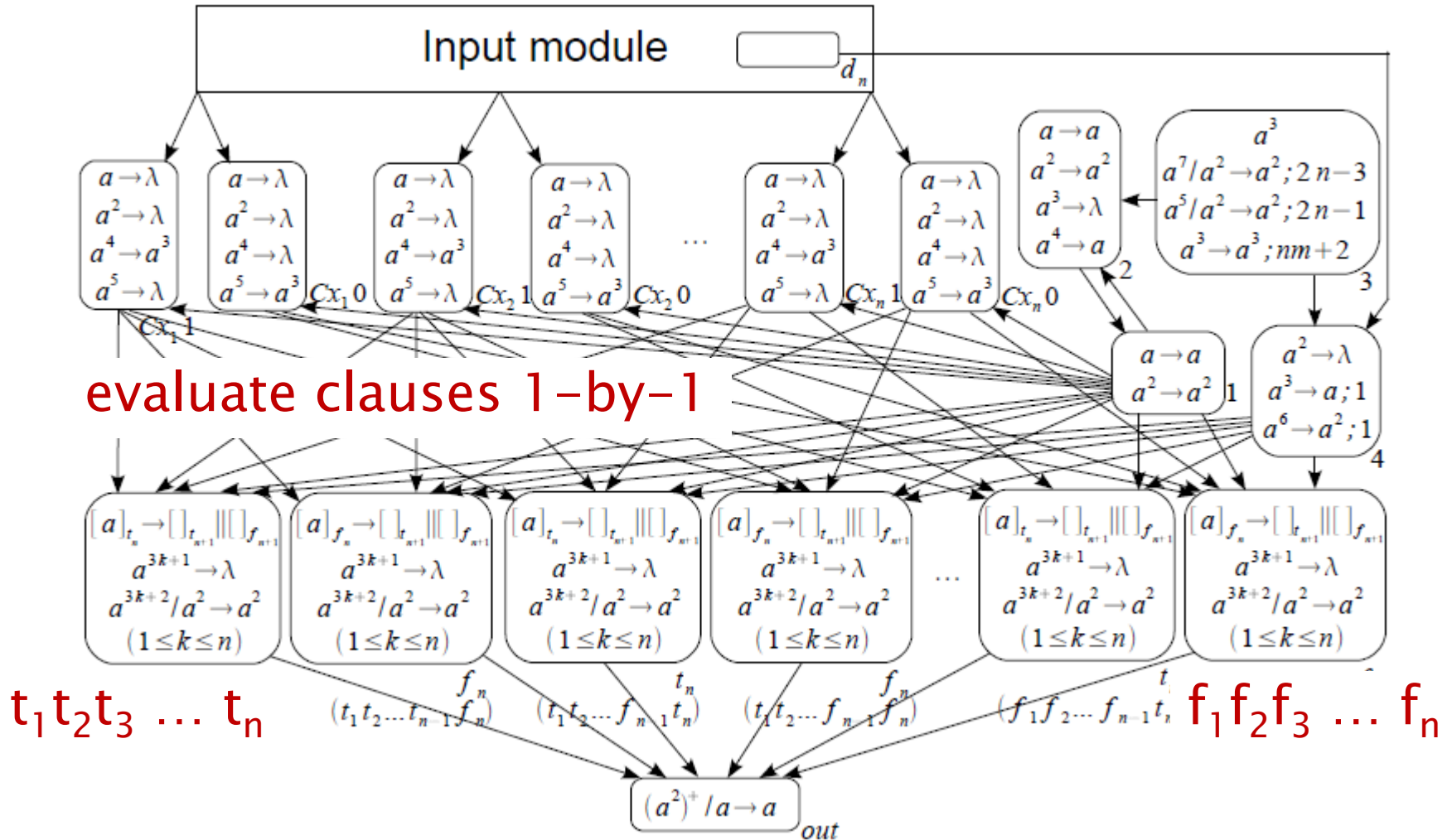
a^3 loop – ‘timing’
for right variable

variable OK add a^3

timing start/stop



satisfiability and output



comparing resources

Resources	this work	“PPP”
Initial number of neurons	11	$4n + 7$
Initial number of spikes	20	9
Number of neuron labels	$10n + 7$	$6n + 8$
Size of synapse dictionary	$6n + 11$	$7n + 6$
Number of rules	$2n^2 + 26n + 26$	$n^2 + 14n + 12$

too much names/labels !
can we reuse & get right structure ?

perhaps improved 'timing' can eliminate labels for consecutive 'linear' steps

division and/or budding

budding → linear size growth so ...

initial system (PPP)

