Cellular Automata and the Quest for Artificial Self-Reproducing Structures

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Overview

- Fundamental Questions
- The Birth of Cellular Automata
- Non-Trivial Self-Reproduction
- Game of Life
- Synchronization
- Signals

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(C) Construction-universality. Can any one, suitably given, automaton be construction-universal, that is, be able to construct every other automaton?

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(E) Evolution. Can the construction of automata by automata progress from simpler types to increasingly complicated types? Also, can this evolution go from less efficient to more efficient automata?

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- What is an artificial construct?
- What is raw material?
- What is an environment?

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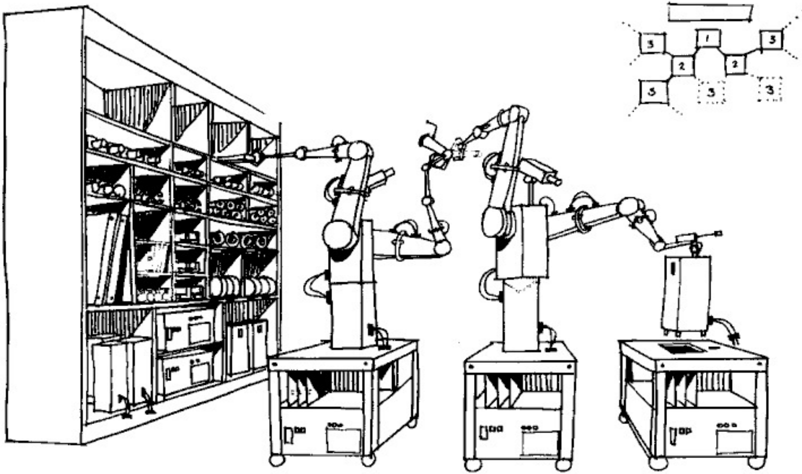
- assemble them into a duplicate of itself,
- and then copy the contents of its memory tape into the empty tape of the offspring.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The Birth of Cellular Automata

Artificial self-reproduction: Abstract from the natural self-reproduction problem its logical form.

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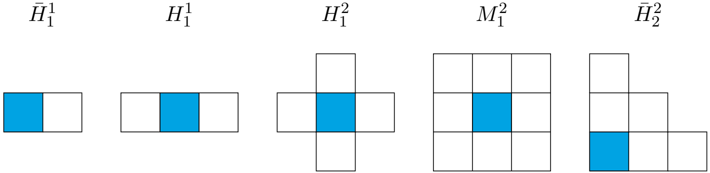
- Two-dimensional grid of cells (machines).
- Synchronous behavior.
- Cells are deterministic finite automata (simplicity).
- All cells are identical (homogeneity).
- One interconnection scheme (homogeneous local communication structures).

Interconnection schemes:



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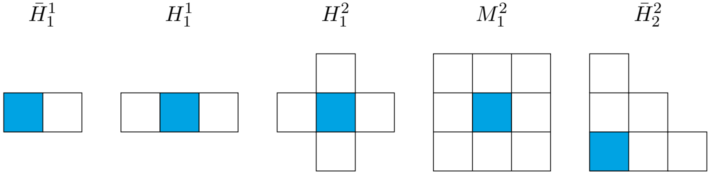

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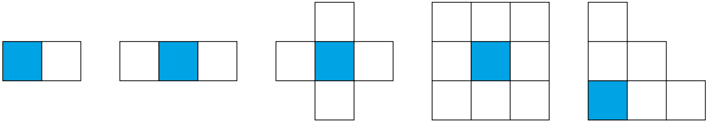
$\bar{H}_1^1$ H_1^1 H_1^2 M_1^2 $\bar{H}_2^2$



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Larger machines are patterns of cell states, embedded in space.

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- The state set is $\{0, 1\}$.
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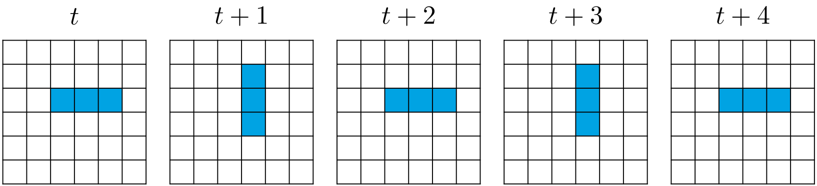
- Each cell is connected to its eight immediate neighbors.
- The local transition function is defined by the sum of the states of the neighbors and of the cell itself. In particular:
- A cell enters state 1, if the sum is three.
- A cell keeps its current state, if the sum is four.
- Otherwise the cell enters state 0.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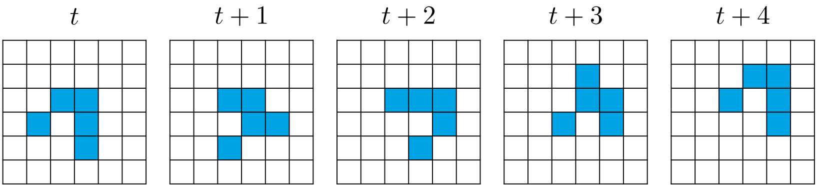
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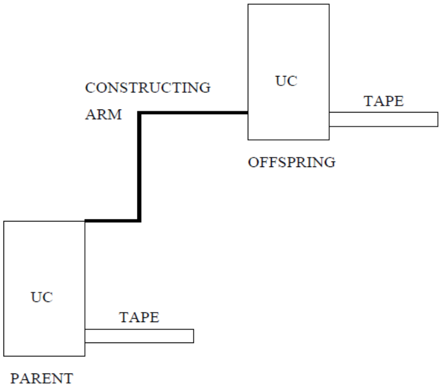
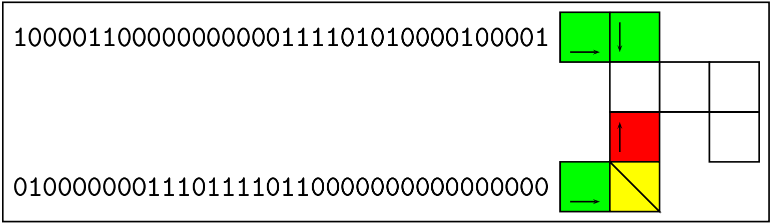
John von Neumann succeeded. He constructed a 29-state cellular automaton which is contruction-universal, self-reproducing, and Turing-universal.





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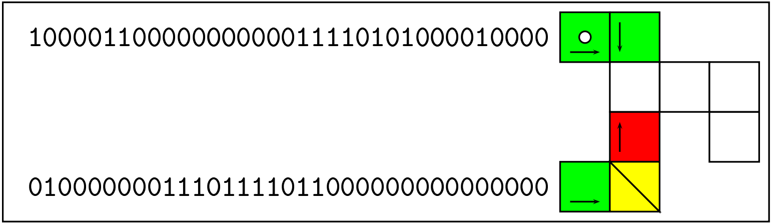

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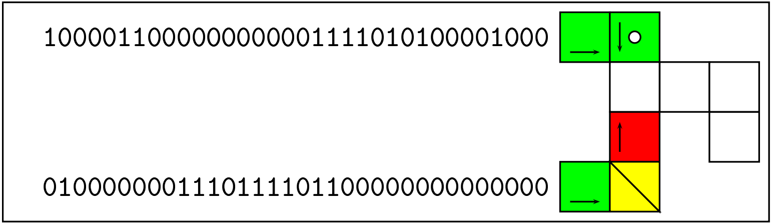
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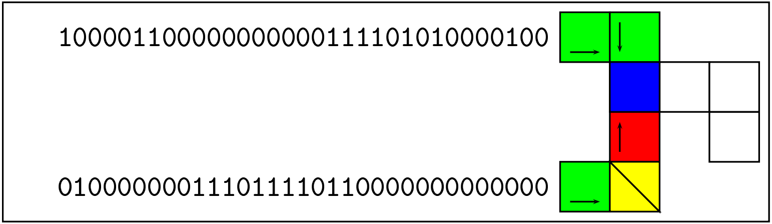

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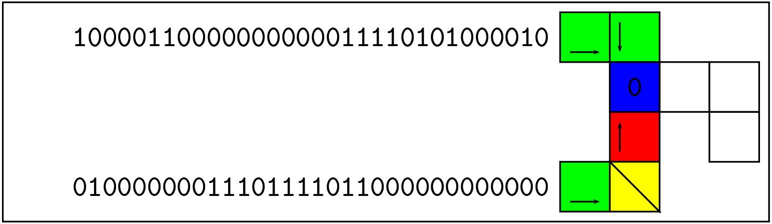
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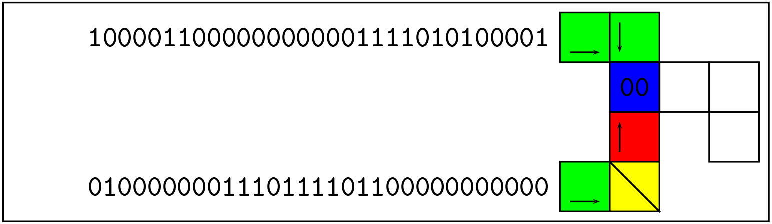
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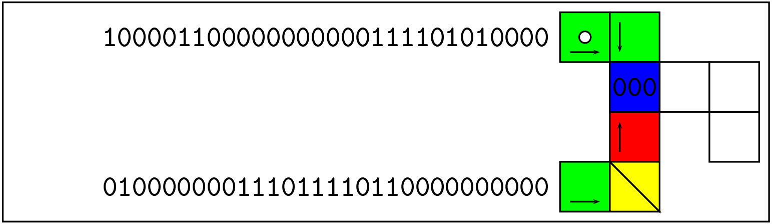

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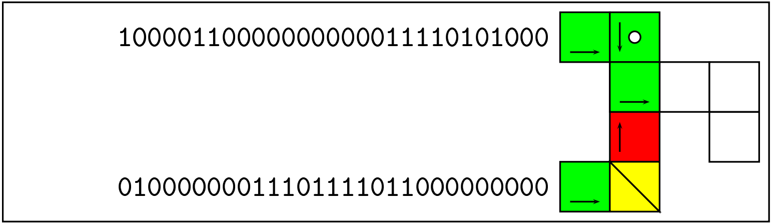
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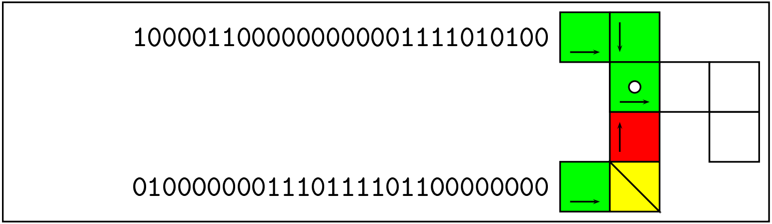

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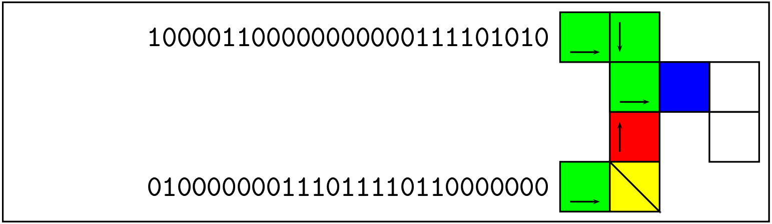

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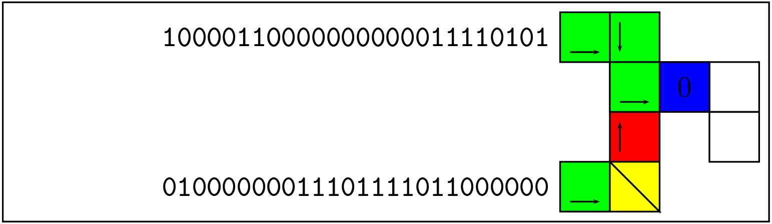
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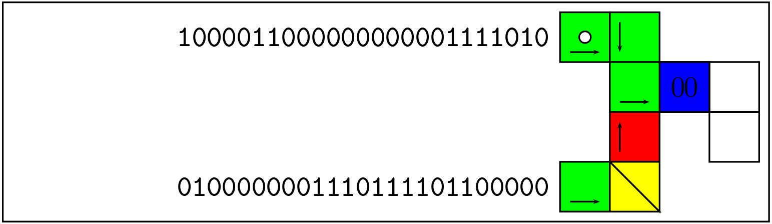

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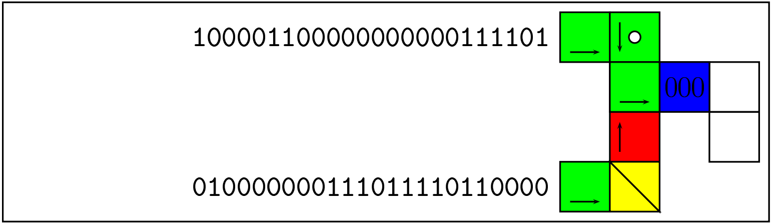
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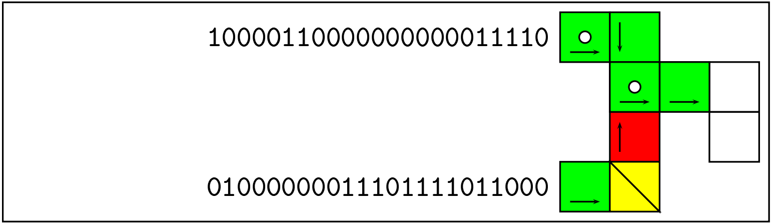
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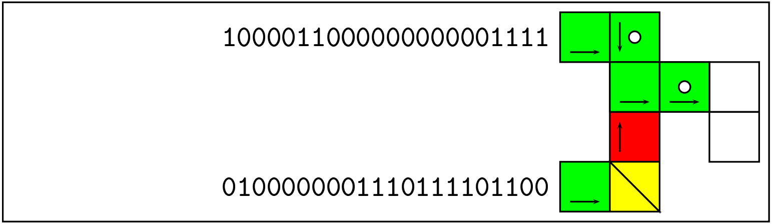
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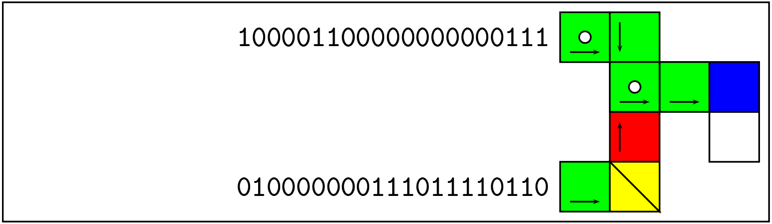

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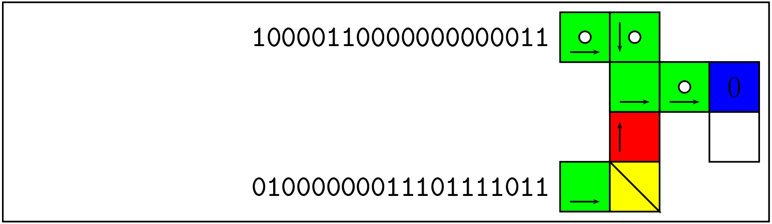
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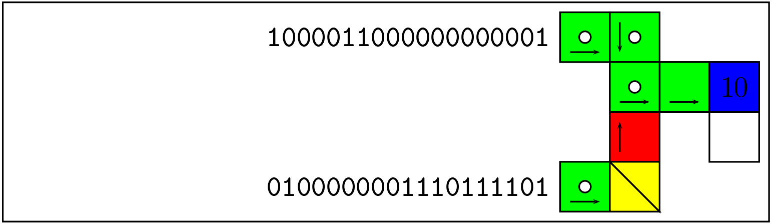
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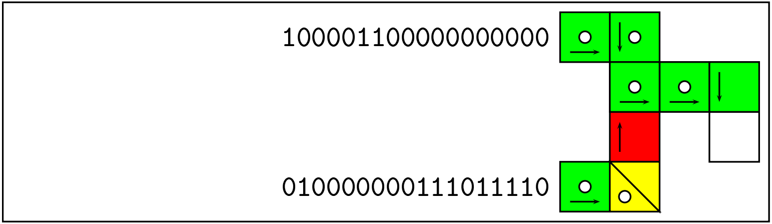
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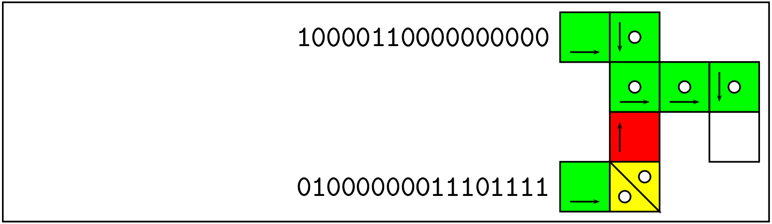

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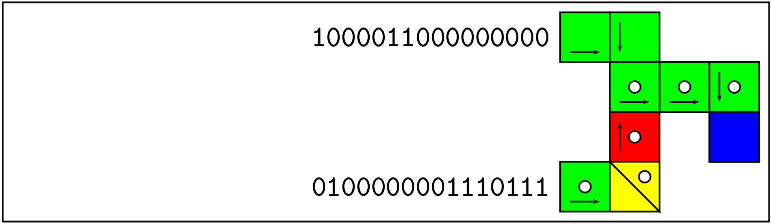
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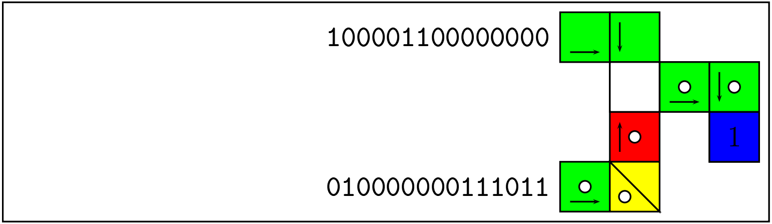
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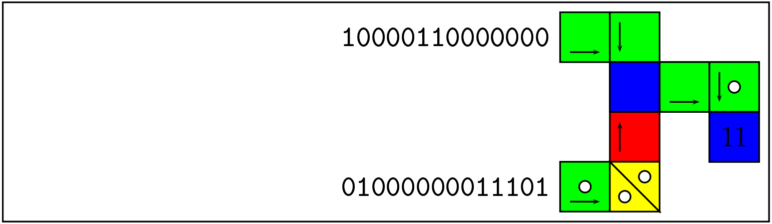
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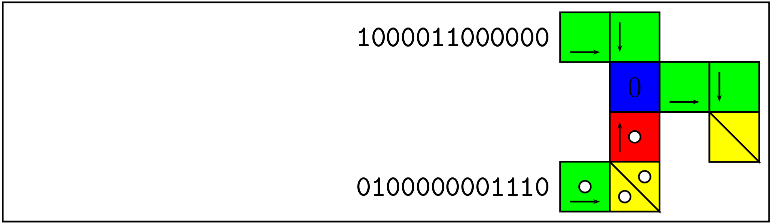
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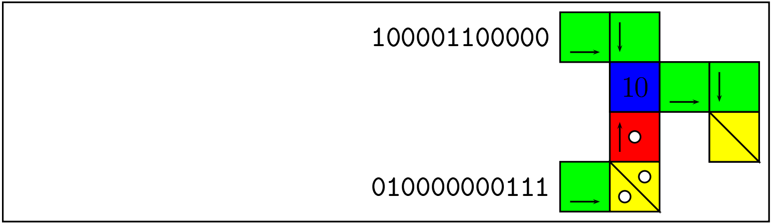
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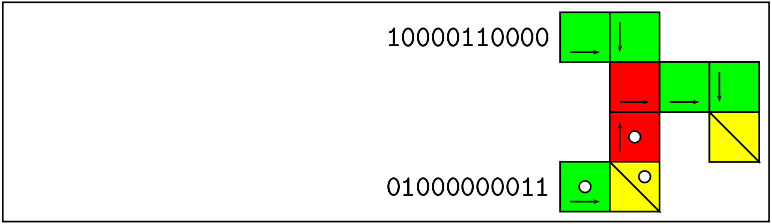

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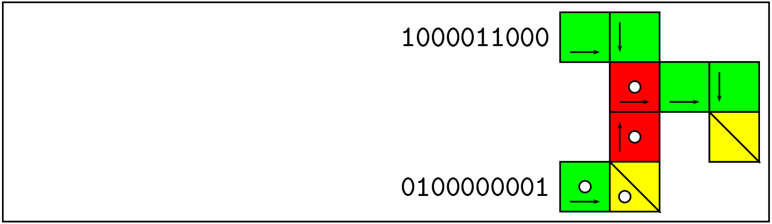
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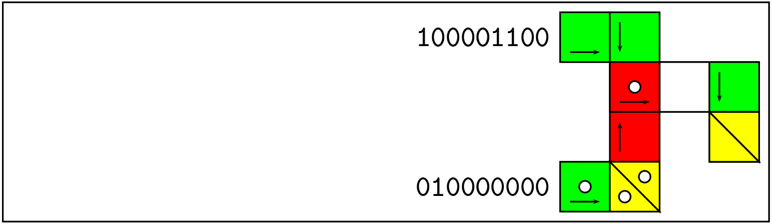

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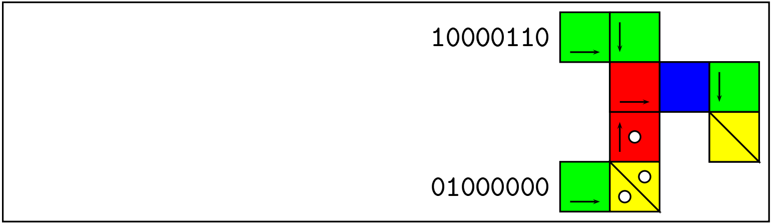
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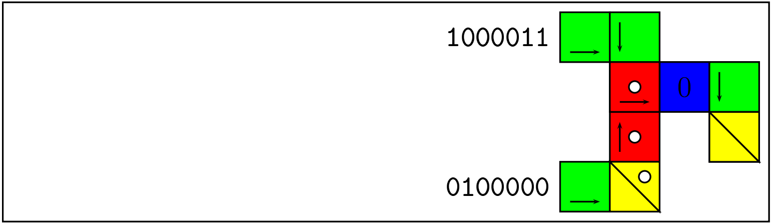
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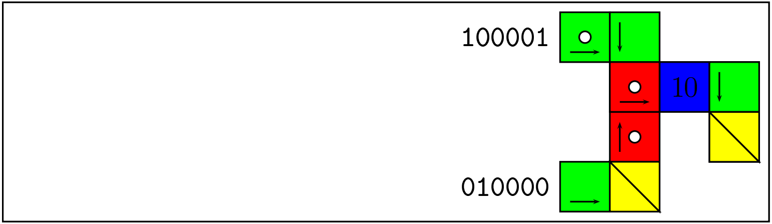
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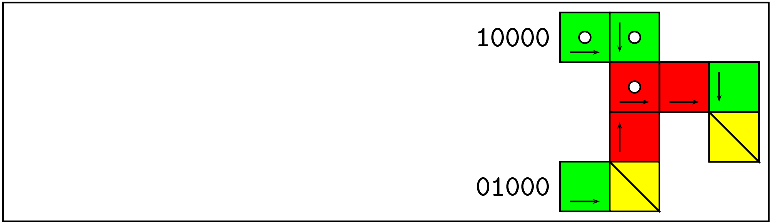
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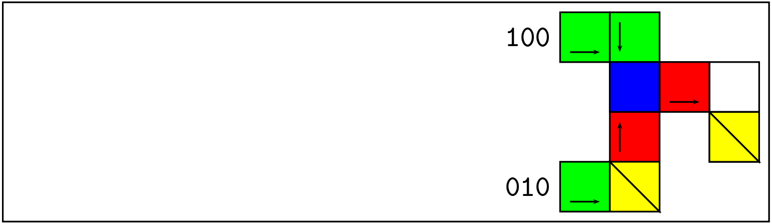
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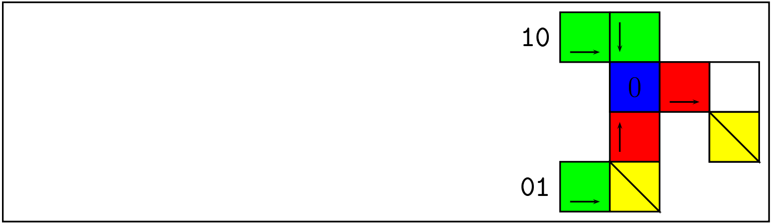
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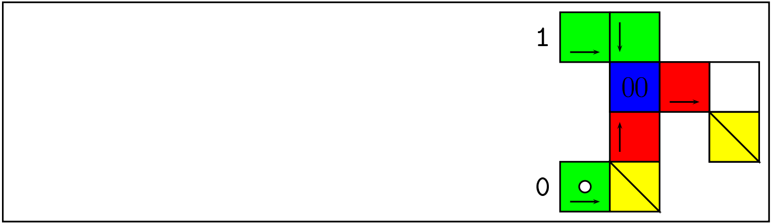
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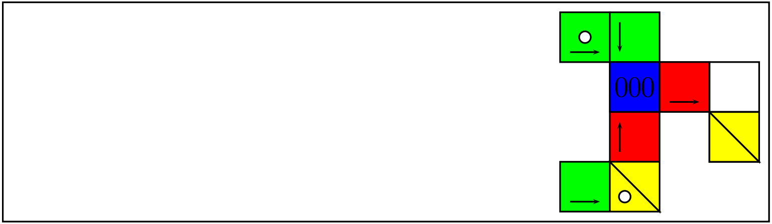
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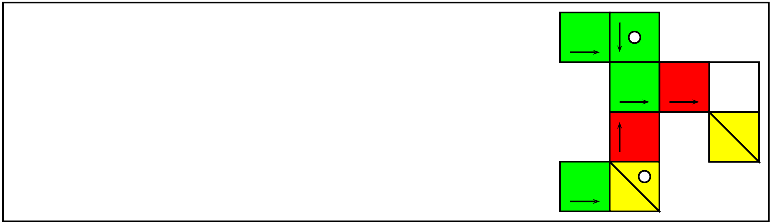

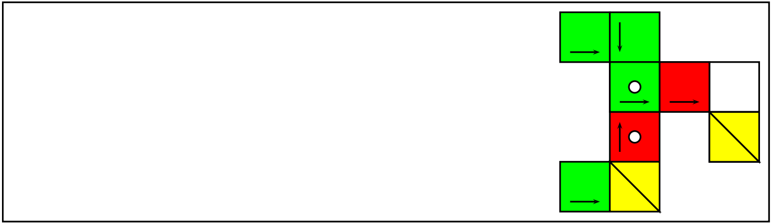

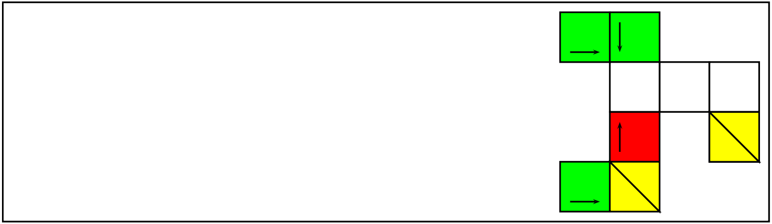

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(D) Self-reproduction. Can a self-reproducing automaton be embedded in von Neumann's 29-state CA? Yes!

Can there be embedded in von Neumann's 29-state CA an automaton which can perform the computations of a universal TM and can also reproduce itself? Yes!

Non-Trivial Self-Reproduction

Observation [Arthur Burks 1970]

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- Given a two-state cellular automaton where a state in the quiescent state changes into the other state if its neighbor to the north is non-quiescent.
- Then a single non-quiescent cell reproduces itself trivially in its neighbor.
- A requirement is needed that the self-reproducing automata have some minimal complexity.
- For example, requiring that the self-reproducing automata are also Turing-universal.

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- Combine trivial, crystaline, self-reproduction with the finite control of a universal Turing machine.

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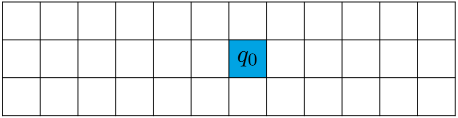

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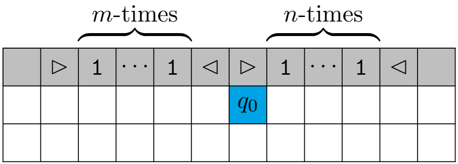

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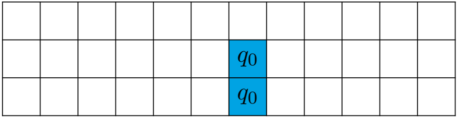
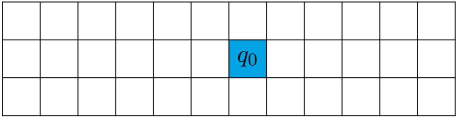


Herman's Cellular Automaton:

- Combine trivial, crystaline, self-reproduction with the finite control of a universal Turing machine.



- What the result does show is that the existence of a self-reproducing universal computer in itself is not relevant to the problem of biological and machine self-reproduction.

Langton's Loop Cellular Automaton:

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```
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```
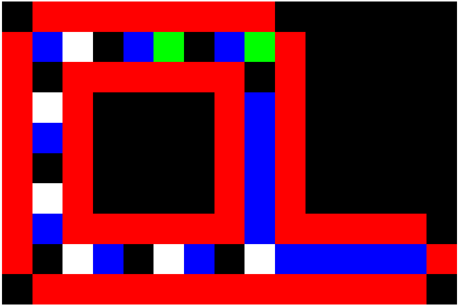
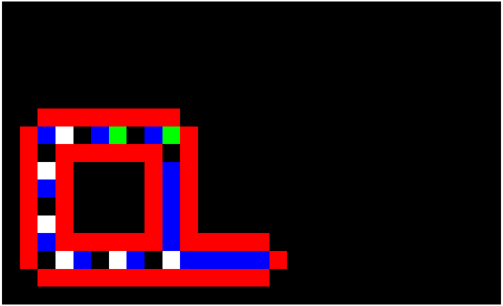

- Signals (genes) travel along the path and are copied at junctions.

```
     212
     212
     212
22222272222222
11061107111111
22222222222222
```

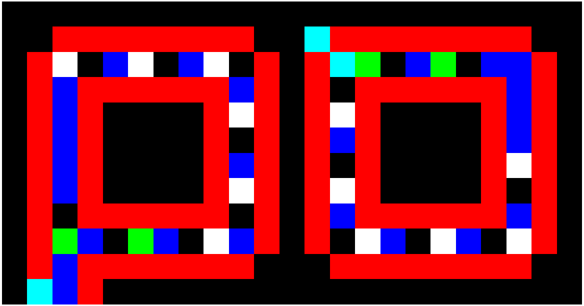
- At the end of the path a gene is interpreted to extend the path.



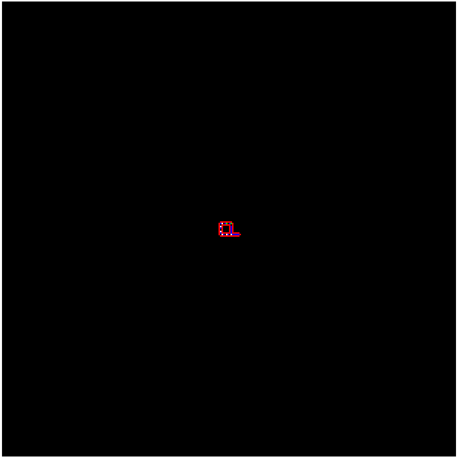
- The offspring is separated by the collision of genes at the new junction.



- If a loop tries to extend its arm to an occupied area, a sheath fragment is generated that absorbes all genes.



- Self-reproduction in infinite space.



So, what is non-trivial self-reproduction?

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John von Neumann 1949

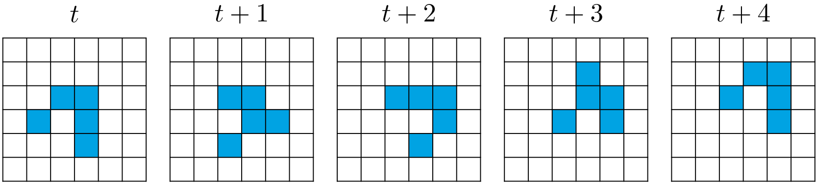
- ... A way around this difficulty [to define non-trivial self-reproduction] is to say that self-reproduction includes the ability to undergo inheritable mutations as well as the ability to make anothe organism like the original ...
- How can a machine manage to construct other machines more "complex" that themselves, in a general and open-ended way – that is, with the potential for unbounded evolutionary growth of complexity.

Game of Life

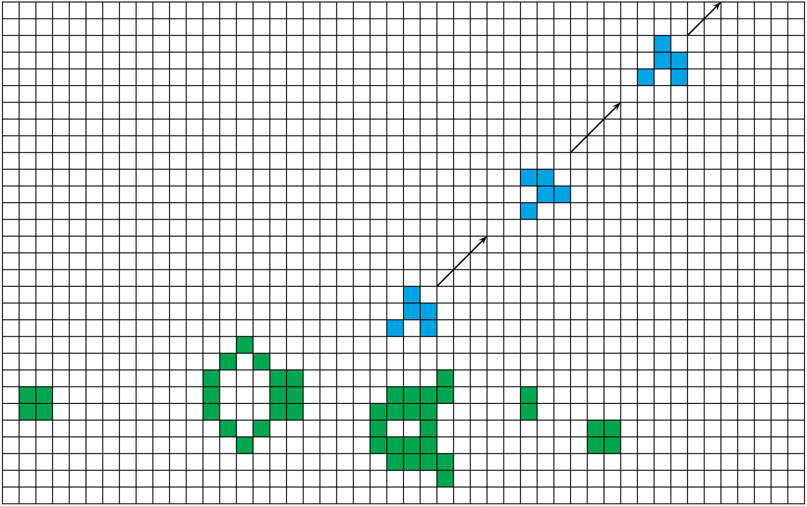
Example revisited:

- The state set is $\{0, 1\}$.
- Each cell is connected to its eight immediate neighbors.

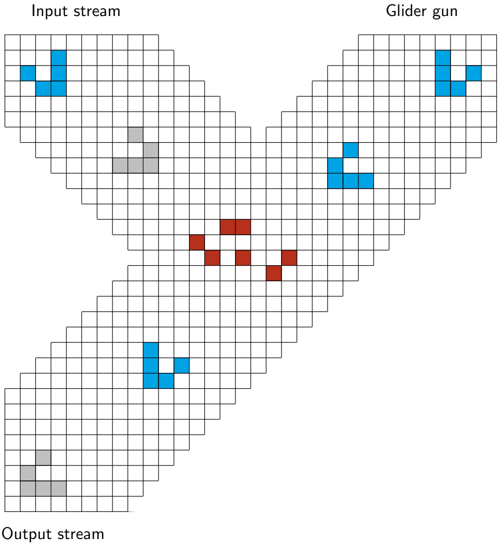
- A cell enters state 1, if the sum of their states is three.
- A cell keeps its current state, if the sum of their states is four.
- Otherwise the cell enters state 0.



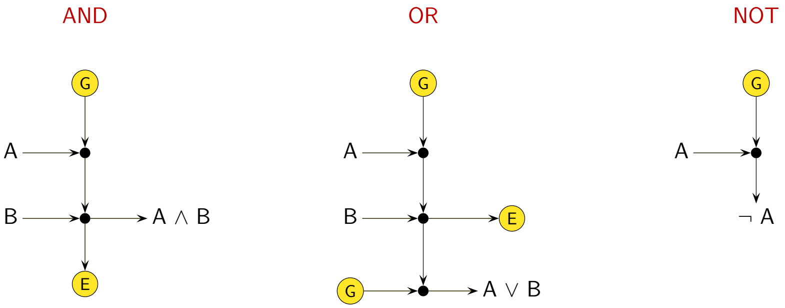
Game of Life: A glider.



Game of Life: A glider gun.



Game of Life: A NOT gate.



Game of Life: Logical gates.

Synchronization

The Firing Squad Synchronization Problem

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Theorem

The minimal solution time for the FSSP is $2n - 2$, where n is the length of the array (chain).

Algorithm

- The problem can be solved by dividing the array in two, four, eight etc. parts of (almost) the same length until all cells are cut-points.
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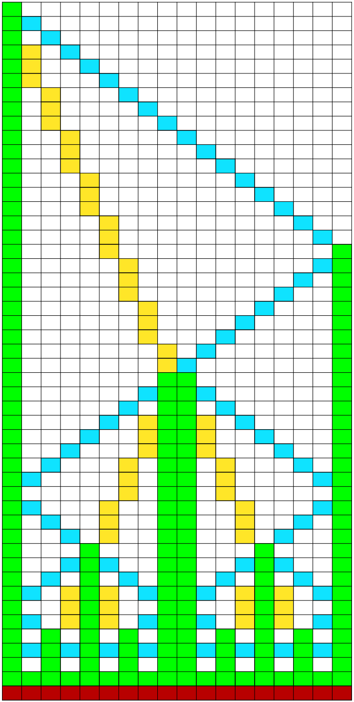
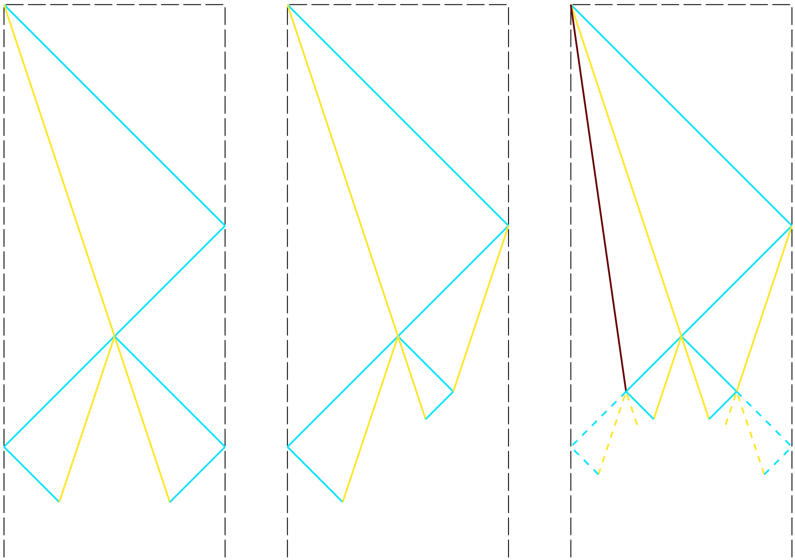
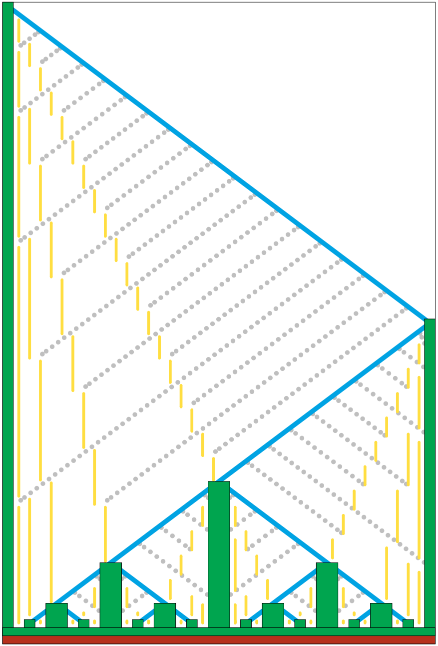
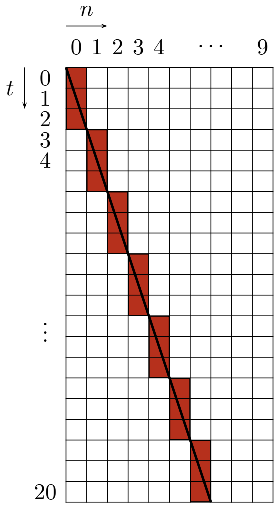
- Exactly at this time the cells will fire synchronously.
- The divisions are performed recursively. At first the array is divided into two parts. Then the process is applied to both parts in parallel, etc.
- In order to divide the array into two parts, the general sends two signals $S1$ and $S2$ to the right.
- Signal $S1$ moves with speed one, and signal $S2$ with speed $1/3$.







Signals



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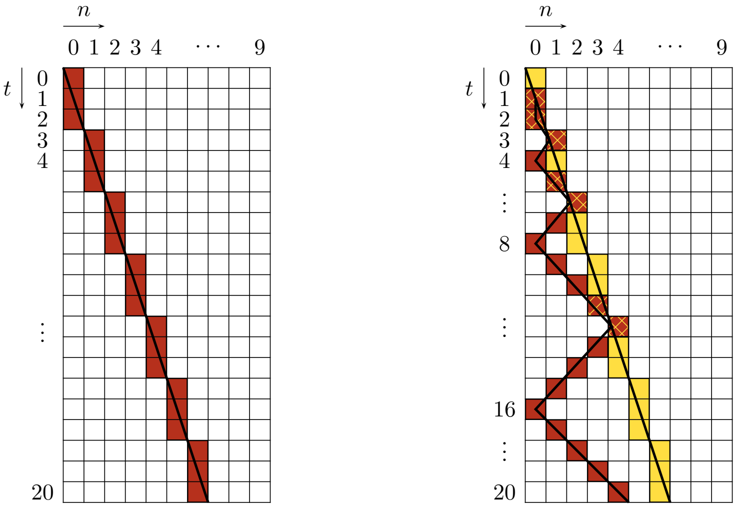
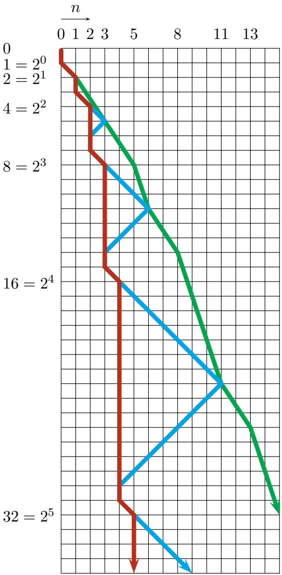
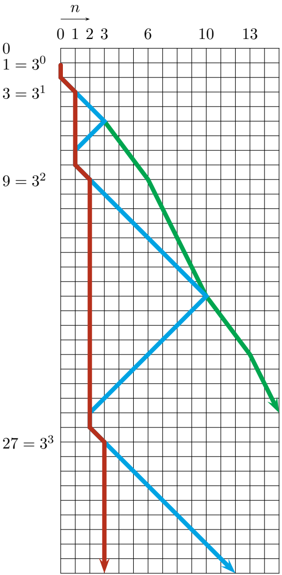
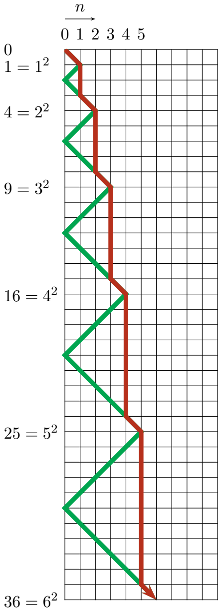


Signals with exponential characteristic function b^n, $b \geq 2$





Signals with polynomial characteristic function n^b, $b \geq 1$



- A signal with characteristic function n^2 can be derived from $(n+1)^2 = n^2 + 2n + 1$.
- In particular, signal ξ has to stay for $2n$ time steps in cell n, and subsequently has to move in one step to cell $n+1$.
- The delay is exactly the time needed by an auxiliary signal that moves from cell n to cell 0 and back.

Signals with polynomial characteristic function n^b, $b \geq 1$

- It holds $(n+1)^3 = n^3 + 3n^2 + 3n + 1$.
- Therefore, a signal with characteristic function n^3 has to stay for $3n^2 + 3n$ time steps in cell n and subsequently has to move in one step to cell $n+1$.
- The delay $3n$ is exactly the time needed by an auxiliary signal that moves from cell n to cell 0 and back and once more to cell 0.
- In cell 0 a modified quadratic signal is generated, which moves from cell 0 to cell n and back and once more to cell n.