Flattening the Transition P Systems with Dissolution

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The general idea

Π **with** m **membranes**	Π^f **with** 1 **membrane**
object a in membrane i	object (a, i)
δ appears in membrane i	object (δ, i)
rule r without "out"	a rule r^f
rule r with "out"	set r^f of rules (possible parents)
membrane i dissoluble	set D_i of rules $(x, i) \to (x, cPar_i)$
mpr step in Π (including communication)	*mpr* step in Π^f using r^f
diss step in Π	*mpr* step in Π^f using D_i
–	special rule $\nabla \to 0$ to ensure separation between applying rules from r^f and rules from D_i

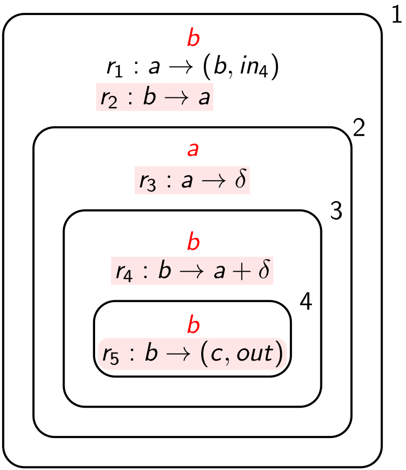
Membrane i is dissoluble: exists $r \in R_i$, $\delta \in rhs(r)$.
Note: i not dissoluble $\Leftrightarrow$ i will not dissolve in any evolution step.

General Notions:

- a rule $u \rightarrow v|_{x_1,\ldots,x_n,\neg y_1,\ldots,y_m}$ has a set of promoters x_i and a set of inhibitors y_j;
- intermediate configuration of a P system of degree m is a vector $W = (w_1, \ldots, w_m)$ with w_i multiset over O or $w_i = *$;
- $w_i = *$ specifies that membrane i has been dissolved;
- $W = (w_1, \ldots, w_m)$ configuration if all $w_i(\delta) = 0$;
- for W, V configurations,

$$W \Longrightarrow_{\mathbf{T}} V \text{ whenever } W \rightarrow_{mpr} V \text{ or } W \rightarrow_{mpr}\rightarrow_{\delta} V$$

Example



1

b

$r_1 : a \rightarrow (b, in_4)$

$r_2 : b \rightarrow a$

2

a

$r_3 : a \rightarrow \delta$

3

b

$r_4 : b \rightarrow a + \delta$

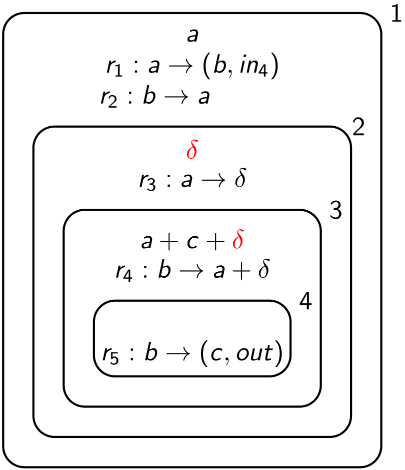
4

b

$r_5 : b \rightarrow (c, out)$

$$(b, a, b, b) \rightarrow_{mpr} (a, \delta, a + c + \delta, 0) \rightarrow_{\delta} (2a + c, *, *, 0) \rightarrow_{mpr}$$
$$\rightarrow_{mpr} (c, *, *, 2b) \rightarrow_{mpr} (3c, *, *, 0)$$

Example



a

$r_1 : a \rightarrow (b, in_4)$

$r_2 : b \rightarrow a$

δ

$r_3 : a \rightarrow \delta$

$a + c + \delta$

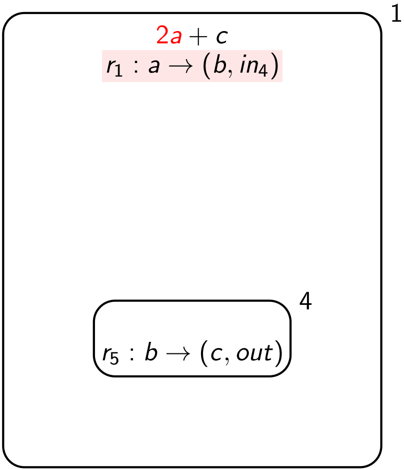
$r_4 : b \rightarrow a + \delta$

$r_5 : b \rightarrow (c, out)$

$$(b, a, b, b) \rightarrow_{mpr} (a, \delta, a + c + \delta, 0) \rightarrow_{\delta} (2a + c, *, *, 0) \rightarrow_{mpr}$$

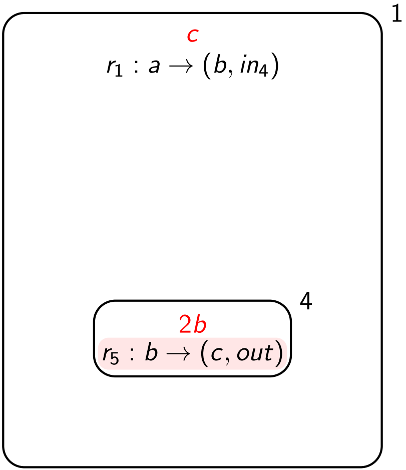
$$\rightarrow_{mpr} (c, *, *, 2b) \rightarrow_{mpr} (3c, *, *, 0)$$

Example



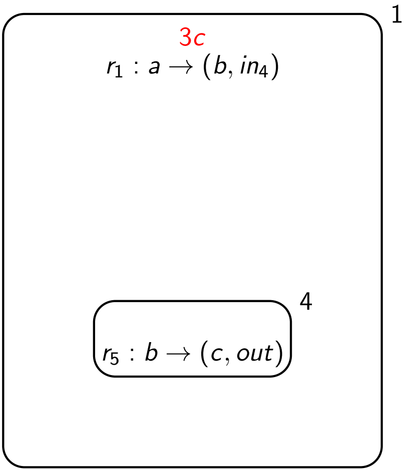
$$(b, a, b, b) \to_{mpr} (a, \delta, a + c + \delta, 0) \to_{\delta} (2a + c, *, *, 0) \to_{mpr}$$
$$\to_{mpr} (c, *, *, 2b) \to_{mpr} (3c, *, *, 0)$$

Example



$$(b, a, b, b) \to_{mpr} (a, \delta, a + c + \delta, 0) \to_{\delta} (2a + c, *, *, 0) \to_{mpr}$$
$$\to_{mpr} (c, *, *, 2b) \to_{mpr} (3c, *, *, 0)$$

Example



1

$3c$

$r_1 : a \to (b, in_4)$

4

$r_5 : b \to (c, out)$

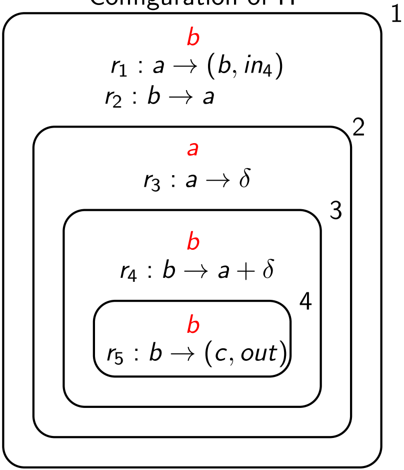
$$(b, a, b, b) \to_{mpr} (a, \delta, a + c + \delta, 0) \to_{\delta} (2a + c, *, *, 0) \to_{mpr}$$
$$\to_{mpr} (c, *, *, 2b) \to_{mpr} (3c, *, *, 0)$$

The flattened P system $\Pi^f = (O^f, \mu^f, R^f)$

- $O^f = (O \cup \{\delta\}) \times \{1, \ldots, m\} \cup \{\nabla\}$;
- μ^f is formed of only one membrane;
- $R^f = \bigcup_{r \in R} r^f \cup \{\nabla \to 0\} \cup \bigcup_{i \text{ dissolvable}} D_i$;

Example (continued)

Configuration of Π

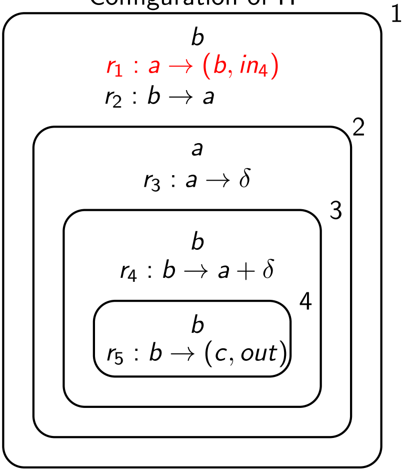


The configuration of Π^f:

$$(b, 1) + (a, 2) + (b, 3) + (b, 4)$$

Example (continued)

Configuration of Π



1

b

$r_1 : a \to (b, in_4)$

$r_2 : b \to a$

2

a

$r_3 : a \to \delta$

3

b

$r_4 : b \to a + \delta$

4

b

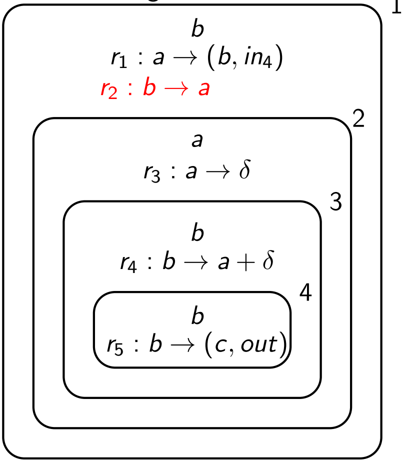
$r_5 : b \to (c, out)$

The set of rules r_1^f contains only one rule:

$$(a, 1) \to (b, 4)|_{(\delta,2),(\delta,3),\neg\nabla}$$

Example (continued)

Configuration of Π



1

b

$r_1 : a \to (b, in_4)$

$r_2 : b \to a$

2

a

$r_3 : a \to \delta$

3

b

$r_4 : b \to a + \delta$

4

b

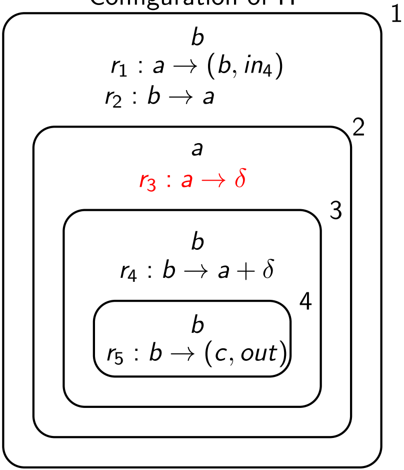
$r_5 : b \to (c, out)$

The set of rules r_2^f also contains only one rule:

$$(b, 1) \to (a, 1)|_{\neg\nabla}$$

Example (continued)

Configuration of Π



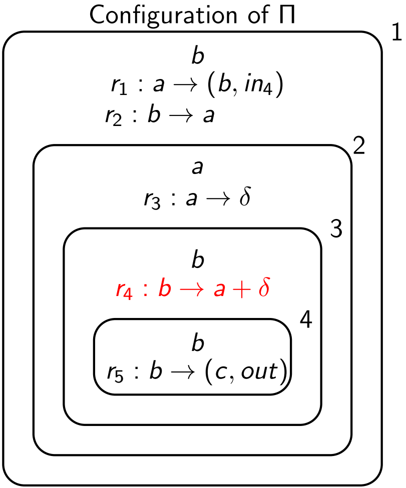
This also takes place for r_3^f:

$$(a, 2) \to (\delta, 2) + \nabla|_{\neg\nabla}$$

and for r_4^f:

$$(b, 3) \to (a + \delta, 3) + \nabla|_{\neg\nabla}$$

Example (continued)



Configuration of Π

1

b

$r_1 : a \to (b, in_4)$

$r_2 : b \to a$

2

a

$r_3 : a \to \delta$

3

b

$r_4 : b \to a + \delta$

4

b

$r_5 : b \to (c, out)$

This also takes place for r_3^f:

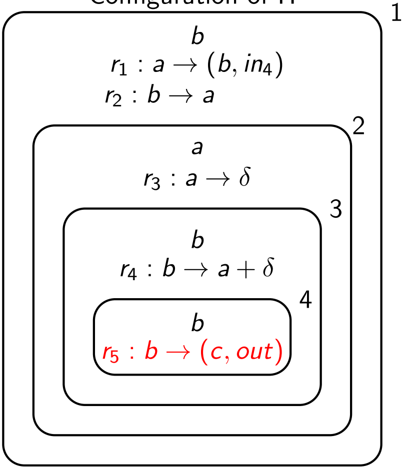
$$(a, 2) \to (\delta, 2) + \nabla|_{\neg\nabla}$$

and for r_4^f:

$$(b, 3) \to (a + \delta, 3) + \nabla|_{\neg\nabla}$$

Example (continued)

Configuration of Π



The set of rules r_5^f contains three rules, one for each possible destination of the c from (c, out):

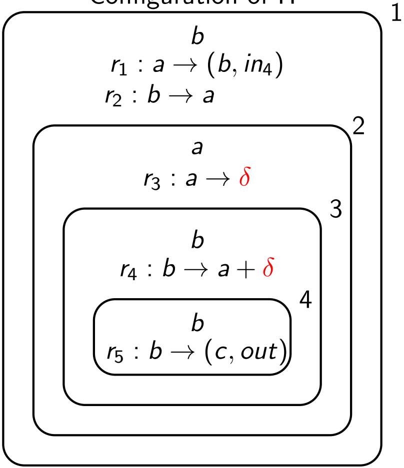
$$(b, 4) \rightarrow (c, 3)|_{\neg\nabla,(\delta,3)}$$

$$(b, 4) \rightarrow (c, 2)|_{(\delta,3),\neg\nabla,(\delta,2)}$$

$$(b, 4) \rightarrow (c, 1)|_{(\delta,3),(\delta,2),\neg\nabla}$$

Example (continued)

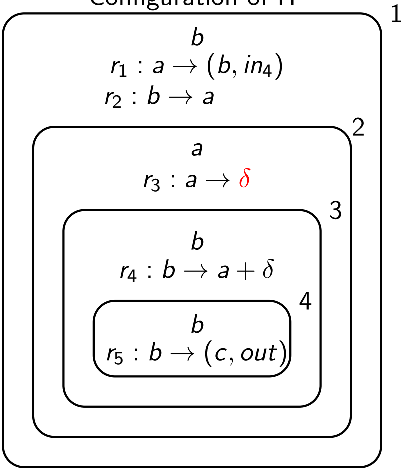
Configuration of Π



We add two sets of rules, D_2 and D_3, to deal with moving objects if membrane 2 or membrane 3 is dissolved.

Example (continued)

Configuration of Π



1

b

$r_1 : a \rightarrow (b, in_4)$

$r_2 : b \rightarrow a$

2

a

$r_3 : a \rightarrow \delta$

3

b

$r_4 : b \rightarrow a + \delta$

4

b

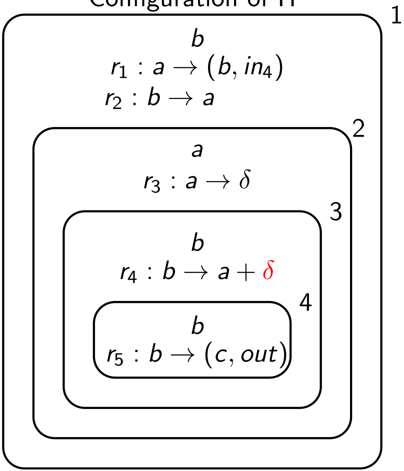
$r_5 : b \rightarrow (c, out)$

The rules in D_2 are given for all $x \in O$:

$$(x, 2) \rightarrow (x, 1)|_{(\delta,2)}$$

Example (continued)

Configuration of Π



b

$r_1 : a \to (b, in_4)$

$r_2 : b \to a$

a

$r_3 : a \to \delta$

b

$r_4 : b \to a + \delta$

b

$r_5 : b \to (c, out)$

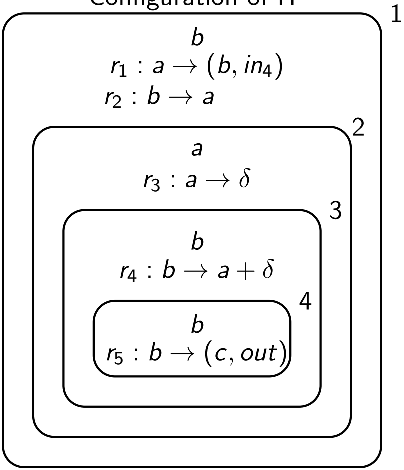
The rules in D_3 are given for all $x \in O$:

$$(x, 3) \to (x, 2)|_{(\delta,3),\neg(\delta,2)}$$

$$(x, 3) \to (x, 1)|_{(\delta,3),(\delta,2)}$$

Example (continued)

Configuration of Π



1

b

$r_1 : a \to (b, in_4)$

$r_2 : b \to a$

2

a

$r_3 : a \to \delta$

3

b

$r_4 : b \to a + \delta$

4

b

$r_5 : b \to (c, out)$

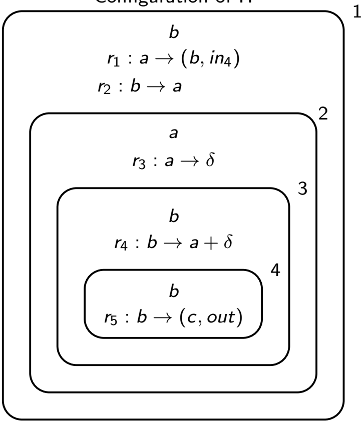
The special symbol ∇ is always produced together with one of the (δ, i) symbols. By appearing, it stops the application of any rule from the sets r^f.
The special rule

$$\nabla \to 0$$

is applied with the rules from sets D_j and by consuming ∇ it allows for rules from r^f to be applied in the next step.

Example (continued)

Configuration of Π



$W_0 = (b, a, b, b)$

Configuration of Π^f

1

$(b,1) + (a,2) + (b,3) + (b,4)$

$r_1^f : (a,1) \to (b,4)|_{(\delta,2),(\delta,3),\neg\nabla}$

$r_2^f : (b,1) \to (a,1)|_{\neg\nabla}$

$r_3^f : (a,2) \to (\delta,2) + \nabla|_{\neg\nabla}$

$r_4^f : (b,3) \to (a+\delta,3) + \nabla|_{\neg\nabla}$

$r_5^f : (b,4) \to (c,3)|_{\neg\nabla,(\delta,3)}$
$(b,4) \to (c,2)|_{(\delta,3),\neg\nabla,(\delta,2)}$
$(b,4) \to (c,1)|_{(\delta,3),(\delta,2),\neg\nabla}$

$\nabla \to 0$

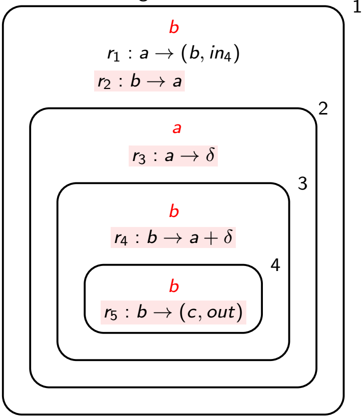
$D_2 : (x,2) \to (x,1)|_{(\delta,2)}$

$D_3 : (x,3) \to (x,2)|_{(\delta,3),\neg(\delta,2)}$
$(x,3) \to (x,1)|_{(\delta,3),(\delta,2)}$

$flat(W_0) = (b,1)+(a,2)+(b,3)+(b,4)$

Example (continued)

Configuration of Π



1

b

$r_1 : a \to (b, in_4)$

$r_2 : b \to a$

2

a

$r_3 : a \to \delta$

3

b

$r_4 : b \to a + \delta$

4

b

$r_5 : b \to (c, out)$

$$(b, a, b, b) \to_{mpr} (a, \delta, a + c + \delta, 0)$$

Configuration of Π^f

1

$(b,1) + (a,2) + (b,3) + (b,4)$

$r_1^f : (a,1) \to (b,4)|_{(\delta,2),(\delta,3),\neg\nabla}$

$r_2^f : (b,1) \to (a,1)|_{\neg\nabla}$

$r_3^f : (a,2) \to (\delta,2) + \nabla|_{\neg\nabla}$

$r_4^f : (b,3) \to (a+\delta,3) + \nabla|_{\neg\nabla}$

$r_5^f : (b,4) \to (c,3)|_{\neg\nabla,(\delta,3)}$
$(b,4) \to (c,2)|_{(\delta,3),\neg\nabla,(\delta,2)}$
$(b,4) \to (c,1)|_{(\delta,3),(\delta,2),\neg\nabla}$

$\nabla \to 0$

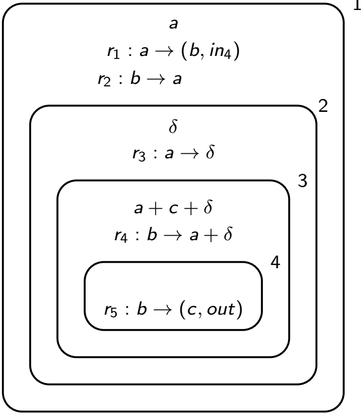
$D_2 : (x,2) \to (x,1)|_{(\delta,2)}$

$D_3 : (x,3) \to (x,2)|_{(\delta,3),\neg(\delta,2)}$
$(x,3) \to (x,1)|_{(\delta,3),(\delta,2)}$

$$(b,1) + (a,2) + (b,3) + (b,4) \\ \to_{mpr} \\ (a,1) + (\delta,2) + (a + c + \delta,3) + 2\nabla$$

Example (continued)

Configuration of Π



$W_1 = (a, \delta, a + c + \delta, 0)$

Configuration of Π^f

1

$(a,1) + (\delta,2) + (a + c + \delta, 3) + 2\nabla$

$r_1^f : (a,1) \to (b,4)|_{(\delta,2),(\delta,3),\neg\nabla}$

$r_2^f : (b,1) \to (a,1)|_{\neg\nabla}$

$r_3^f : (a,2) \to (\delta,2) + \nabla|_{\neg\nabla}$

$r_4^f : (b,3) \to (a+\delta,3) + \nabla|_{\neg\nabla}$

$r_5^f : (b,4) \to (c,3)|_{\neg\nabla,(\delta,3)}$
$(b,4) \to (c,2)|_{(\delta,3),\neg\nabla,(\delta,2)}$
$(b,4) \to (c,1)|_{(\delta,3),(\delta,2),\neg\nabla}$

$\nabla \to 0$

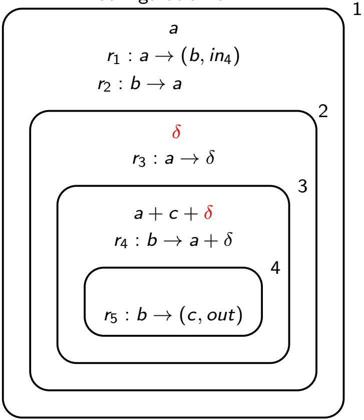
$D_2 : (x,2) \to (x,1)|_{(\delta,2)}$

$D_3 : (x,3) \to (x,2)|_{(\delta,3),\neg(\delta,2)}$
$(x,3) \to (x,1)|_{(\delta,3),(\delta,2)}$

$flat(W_1) =$
$(a,1) + (\delta,2) + (a + c + \delta, 3) + 2\nabla$

Example (continued)

Configuration of Π



$$(a, \delta, a + c + \delta, 0) \to_{\delta} (2a + c, *, *, 0)$$

Configuration of Π^f

1

$$(a, 1) + (\delta, 2) + (a + c + \delta, 3) + 2\nabla$$

$r_1^f : (a, 1) \to (b, 4)|_{(\delta,2),(\delta,3),\neg\nabla}$

$r_2^f : (b, 1) \to (a, 1)|_{\neg\nabla}$

$r_3^f : (a, 2) \to (\delta, 2) + \nabla|_{\neg\nabla}$

$r_4^f : (b, 3) \to (a + \delta, 3) + \nabla|_{\neg\nabla}$

$r_5^f : (b, 4) \to (c, 3)|_{\neg\nabla,(\delta,3)}$
$(b, 4) \to (c, 2)|_{(\delta,3),\neg\nabla,(\delta,2)}$
$(b, 4) \to (c, 1)|_{(\delta,3),(\delta,2),\neg\nabla}$

$\nabla \to 0$

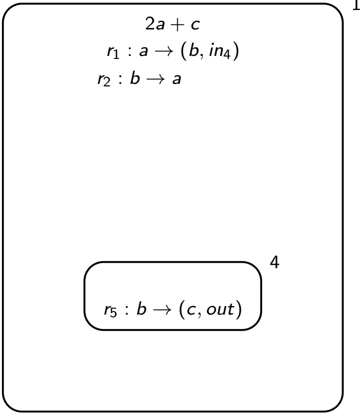
$D_2 : (x, 2) \to (x, 1)|_{(\delta,2)}$

$D_3 : (x, 3) \to (x, 2)|_{(\delta,3),\neg(\delta,2)}$
$(x, 3) \to (x, 1)|_{(\delta,3),(\delta,2)}$

$$(a, 1) + (\delta, 2) + (a + c + \delta, 3) + 2\nabla \to_{mpr} (2a + c, 1) + (\delta, 2) + (\delta, 3)$$

Example (continued)

Configuration of Π



$2a + c$

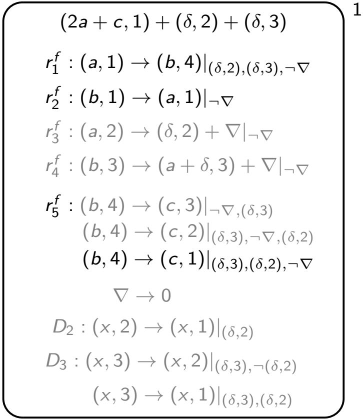
$r_1 : a \to (b, in_4)$

$r_2 : b \to a$

$r_5 : b \to (c, out)$

$W_2 = (2a + c, *, *, 0)$

Configuration of Π^f



$(2a + c, 1) + (\delta, 2) + (\delta, 3)$

$r_1^f : (a, 1) \to (b, 4)|_{(\delta,2),(\delta,3),\neg\nabla}$

$r_2^f : (b, 1) \to (a, 1)|_{\neg\nabla}$

$r_3^f : (a, 2) \to (\delta, 2) + \nabla|_{\neg\nabla}$

$r_4^f : (b, 3) \to (a + \delta, 3) + \nabla|_{\neg\nabla}$

$r_5^f : (b, 4) \to (c, 3)|_{\neg\nabla,(\delta,3)}$
$(b, 4) \to (c, 2)|_{(\delta,3),\neg\nabla,(\delta,2)}$
$(b, 4) \to (c, 1)|_{(\delta,3),(\delta,2),\neg\nabla}$

$\nabla \to 0$

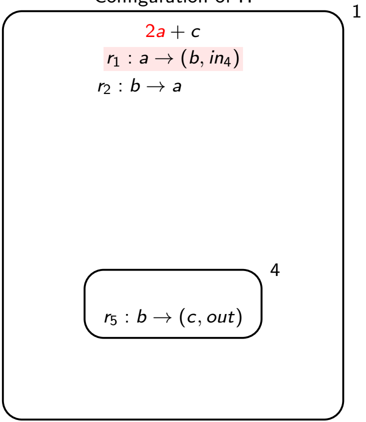
$D_2 : (x, 2) \to (x, 1)|_{(\delta,2)}$

$D_3 : (x, 3) \to (x, 2)|_{(\delta,3),\neg(\delta,2)}$
$(x, 3) \to (x, 1)|_{(\delta,3),(\delta,2)}$

$flat(W_2) = (2a + c, 1) + (\delta, 2) + (\delta, 3)$

Example (continued)

Configuration of Π



$2a + c$

$r_1 : a \to (b, in_4)$

$r_2 : b \to a$

$r_5 : b \to (c, out)$

$(2a + c, *, *, 0) \to_{mpr} (c, *, *, 2b)$

Configuration of Π^f

1

$(2a + c, 1) + (\delta, 2) + (\delta, 3)$

$r_1^f : (a, 1) \to (b, 4)|_{(\delta,2),(\delta,3),\neg\nabla}$

$r_2^f : (b, 1) \to (a, 1)|_{\neg\nabla}$

$r_3^f : (a, 2) \to (\delta, 2) + \nabla|_{\neg\nabla}$

$r_4^f : (b, 3) \to (a + \delta, 3) + \nabla|_{\neg\nabla}$

$$\begin{aligned} r_5^f : & (b, 4) \to (c, 3)|_{\neg\nabla,(\delta,3)} \\ & (b, 4) \to (c, 2)|_{(\delta,3),\neg\nabla,(\delta,2)} \\ & (b, 4) \to (c, 1)|_{(\delta,3),(\delta,2),\neg\nabla} \end{aligned}$$

$\nabla \to 0$

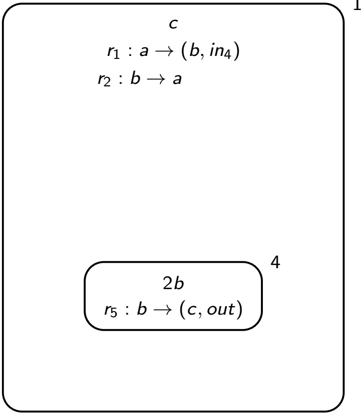
$D_2 : (x, 2) \to (x, 1)|_{(\delta,2)}$

$$\begin{aligned} D_3 : & (x, 3) \to (x, 2)|_{(\delta,3),\neg(\delta,2)} \\ & (x, 3) \to (x, 1)|_{(\delta,3),(\delta,2)} \end{aligned}$$

$$(2a + c, 1) + (\delta, 2) + (\delta, 3) \\ \to_{mpr} \\ (c, 1) + (\delta, 2) + (\delta, 3) + (2b, 4)$$

Example (continued)

Configuration of Π



1

c

$r_1 : a \to (b, in_4)$

$r_2 : b \to a$

4

$2b$

$r_5 : b \to (c, out)$

$W_3 = (c, *, *, 2b)$

Configuration of Π^f

1

$(c, 1) + (\delta, 2) + (\delta, 3) + (2b, 4)$

$r_1^f : (a, 1) \to (b, 4)|_{(\delta,2),(\delta,3),\neg\nabla}$

$r_2^f : (b, 1) \to (a, 1)|_{\neg\nabla}$

$r_3^f : (a, 2) \to (\delta, 2) + \nabla|_{\neg\nabla}$

$r_4^f : (b, 3) \to (a + \delta, 3) + \nabla|_{\neg\nabla}$

$r_5^f : (b, 4) \to (c, 3)|_{\neg\nabla,(\delta,3)}$
$(b, 4) \to (c, 2)|_{(\delta,3),\neg\nabla,(\delta,2)}$
$(b, 4) \to (c, 1)|_{(\delta,3),(\delta,2),\neg\nabla}$

$\nabla \to 0$

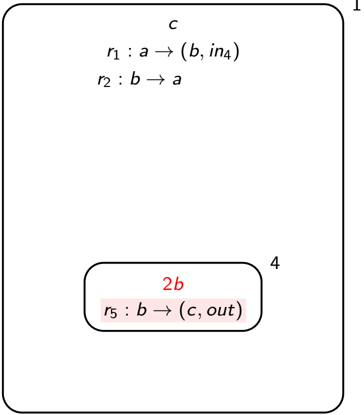
$D_2 : (x, 2) \to (x, 1)|_{(\delta,2)}$

$D_3 : (x, 3) \to (x, 2)|_{(\delta,3),\neg(\delta,2)}$
$(x, 3) \to (x, 1)|_{(\delta,3),(\delta,2)}$

$flat(W_3) = (c, 1) + (\delta, 2) + (\delta, 3) + (2b, 4)$

Example (continued)

Configuration of Π



1

c

$r_1 : a \to (b, in_4)$

$r_2 : b \to a$

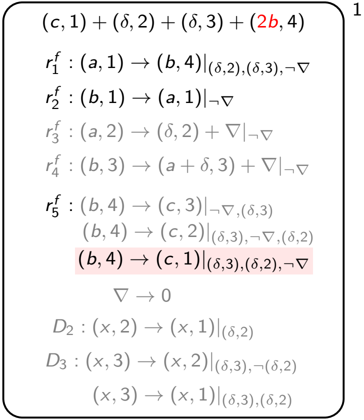
4

$2b$

$r_5 : b \to (c, out)$

$$(c, *, *, 2b) \to_{mpr} (3c, *, *, 0)$$

Configuration of Π^f



1

$$(c, 1) + (\delta, 2) + (\delta, 3) + (2b, 4)$$

$$r_1^f : (a, 1) \to (b, 4)|_{(\delta,2),(\delta,3),\neg\nabla}$$

$$r_2^f : (b, 1) \to (a, 1)|_{\neg\nabla}$$

$$r_3^f : (a, 2) \to (\delta, 2) + \nabla|_{\neg\nabla}$$

$$r_4^f : (b, 3) \to (a + \delta, 3) + \nabla|_{\neg\nabla}$$

$$r_5^f : (b, 4) \to (c, 3)|_{\neg\nabla,(\delta,3)}$$

$$(b, 4) \to (c, 2)|_{(\delta,3),\neg\nabla,(\delta,2)}$$

$$(b, 4) \to (c, 1)|_{(\delta,3),(\delta,2),\neg\nabla}$$

$$\nabla \to 0$$

$$D_2 : (x, 2) \to (x, 1)|_{(\delta,2)}$$

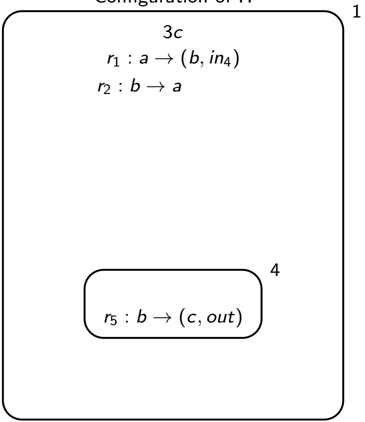
$$D_3 : (x, 3) \to (x, 2)|_{(\delta,3),\neg(\delta,2)}$$

$$(x, 3) \to (x, 1)|_{(\delta,3),(\delta,2)}$$

$$(c, 1) + (\delta, 2) + (\delta, 3) + (2b, 4) \to_{mpr} (3c, 1) + (\delta, 2) + (\delta, 3)$$

Example (continued)

Configuration of Π



$$W_4 = (3c, *, *, 0)$$

Configuration of Π^f

1

$$(3c, 1) + (\delta, 2) + (\delta, 3)$$

$r_1^f : (a, 1) \to (b, 4)|_{(\delta,2),(\delta,3),\neg\nabla}$

$r_2^f : (b, 1) \to (a, 1)|_{\neg\nabla}$

$r_3^f : (a, 2) \to (\delta, 2) + \nabla|_{\neg\nabla}$

$r_4^f : (b, 3) \to (a + \delta, 3) + \nabla|_{\neg\nabla}$

$r_5^f : (b, 4) \to (c, 3)|_{\neg\nabla,(\delta,3)}$
$(b, 4) \to (c, 2)|_{(\delta,3),\neg\nabla,(\delta,2)}$
$(b, 4) \to (c, 1)|_{(\delta,3),(\delta,2),\neg\nabla}$

$\nabla \to 0$

$D_2 : (x, 2) \to (x, 1)|_{(\delta,2)}$

$D_3 : (x, 3) \to (x, 2)|_{(\delta,3),\neg(\delta,2)}$
$(x, 3) \to (x, 1)|_{(\delta,3),(\delta,2)}$

$$flat(W_4) = (3c, 1) + (\delta, 2) + (\delta, 3)$$

Main Results

Theorem

W and V configurations of Π:

$$\textbf{if } W \Longrightarrow_{\mathbf{T}} V \textbf{ then } \mathit{flat}(W) \Longrightarrow_{\mathbf{T}} \mathit{flat}(V) \textit{ or } \mathit{flat}(W) \Longrightarrow_{\mathbf{T}}\Longrightarrow_{\mathbf{T}} \mathit{flat}(V)$$

$$\textbf{if } \mathit{flat}(W) \Longrightarrow_{\mathbf{T}} \mathit{flat}(V) \textbf{ then } W \Longrightarrow_{\mathbf{T}} V$$

Corollary

If W, V configurations of Π which has no dissolutions then

$$W \Longrightarrow_{\mathbf{T}} V \text{ if and only if } \mathit{flat}(W) \Longrightarrow_{\mathbf{T}} \mathit{flat}(V)$$

Conclusion

- semantically conservative transformation of arbitrary transition P systems with dissolution into a P system with one membrane;
- one evolution step in $\Pi \longleftrightarrow$ one or two evolution steps in Π^f;
- based on a simple semantics which does not require additional syntax for the flat form;
- can be used to reduce problems for P systems with multiple membranes to simpler cases (one membrane).

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Thank you for listening!